TOOLS FOR APPLIED MACRO 2017

PROBLEM SET 1

Instructions

Write up your answers carefully. Hand in during class on Wednesday February 15.

Questions

(1) Find the autocovariance function

$$\gamma_j \equiv E\left(x_t x_{t-j}\right)$$

for the MA(2) process

$$x_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} : \varepsilon_t \sim N(0,1)$$

and $j = 0, 1, 2, 3$.

(2) Find the $MA(\infty)$ representation of the AR(1) process

$$y_t = \rho y_{t-1} + \varepsilon_t : \varepsilon_t \sim N(0,1)$$

(3) Consider the (Hilbert) space of square summable sequences l^2 defined as the space of all sequences $\{a_0, a_1, a_2, ...\}$ such that

$$\sum_{j=0}^{\infty} a_j^2 < \infty$$

Show that

$$\langle a, b \rangle \equiv \sum_{j=0}^{\infty} a_j b_j$$

is a valid inner product and that

$$\|a\| \equiv \sqrt{\sum_{j=0}^{\infty} a_j^2}$$

is a valid norm.

(4) Use that the sequence of coefficients in an $MA(\infty)$ process with finite variance can be considered an element of l^2. What is then the inner product $\langle x_t, x_{t-2} \rangle$ if x_t is defined as in Q1?

(5) Find the distance (or metric) $\|x_t - y_t\|$ where x_t and y_t are defined as in Q1 and Q2. What object does the distance $\|x_t - y_t\|$ correspond to?

Date: February 8, 2017.

(6) For l^2, the projection theorem states that $\widehat{y}_t$ is the minimum variance estimate of y_t conditional on y_{t-1} iff $\langle y_t - \widehat{y}_t, y_{t-1} \rangle = 0$. Find the scalar g such that $\widehat{y}_t = g y_{t-1}$and $\langle y_t - \widehat{y}_t, y_{t-1} \rangle = 0$ by explicitly computing the inner products. Compare to the expression for y_t above.

(7) Assume that the ε_ts in Q1 and Q2 are the same. Find the scalar k as a function of the θs and ρ such that $\widehat{x}_t \equiv k y_t$ is the minimum variance estimate of x_t conditional on y_t.