TOPICS IN MACROECONOMICS: MODELING INFORMATION, LEARNING AND EXPECTATIONS

EXAM JUNE 19 2012

There are 4 questions but you only need to answer 3 of them. Each answered question is worth a maximum of 10 points. Put your name on each sheet of paper that you hand in. Write clearly. Number the pages. Include a "front page" with your name, which questions you answered and how many pages you handed in. Sign the front page. Good luck.

Question 1: The Lucas Island model

Consider the Lucas island model. Supply $y_t(z)$ on island z is given by

$$y_t(z) = \gamma \left[P_t(z) - E\left(P_t \mid I_t(z)\right)\right]$$

where the price $P_t(z)$ on island z is (exogenously) given by

$$P_t(z) = P_t + z_t : z_t \sim N(0, \tau^2)$$

and where P_t is the aggregate price level and $I_t(z)$ is the information set of island z inhabitants.

a) Find supply on island z as a function of $P_t(z)$ and the (common across islands) prior $\overline{P}$ defined as the conditional mean of distribution of the the aggregate price level P_t

$$P_t \sim N\left(\overline{P}_t, \sigma^2\right)$$

Interpret.

b) (The log of) nominal demand is postulated as

$$\begin{aligned} y_t + P_t &= x_t \\ \Delta x_t &\equiv (x_t - x_{t-1}) \sim N\left(0, \sigma_x^2\right) \end{aligned}$$

Solve for real output and the aggregate price level as a function of x_t and x_{t-1}.

c) How does the variance of inflation and output depend on the parameters γ and τ? (You may treat σ^2 as given.)

d) Lucas assumed that the information was pooled across islands between periods. Lorenzoni (AER 2009) proposes that noisy public signals can have effects akin to demand shocks in a modern version of a Lucas type island economy. What does Lorenzoni (2009) assume about information sharing? How does Lorenzoni ensure that trading does not reveal all aggregate information perfectly?

Question 2: Endogenous Information Choice and Rational Inattention

a) Let the entropy of the n-dimensional vector X be $h(X)$ and let $Y = BX$ with $rank(B) = n$. What is the conditional entropy $h(X \mid Y)$? What is the mutual information $I(X;Y)$?

b) For a uniformly distributed variable $\theta \sim U(0,1)$ what is the channel capacity (measured in bits) needed to ensure that the posterior error is smaller than 0.5? Smaller than 0.25?

c) For a uniformly distributed variable $\theta \sim U(a,b)$ what is the channel capacity needed to ensure that the posterior error is smaller than ε?

d) Solve for the optimal allocation of attention (i.e. choose posterior variances) in the following set up:

Expected loss

$$EU = \lambda^2\sigma_1^2 + (1-\lambda)^2\sigma_2^2$$

subject to

$$\ln|\Sigma_{prior}| - \ln|\Sigma_{post}| \leq \kappa$$

where

$$\Sigma_{post} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

for

$$\begin{aligned} \Sigma_{prior} &= \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \\ e^{-\kappa} &= 1/10 \end{aligned}$$

e) Define and explain the "no forgetting constraint".

f) Find two marginal conditions for λ where the "no forgetting constraint" starts/stops binding.

Question 3: Private and public information

Consider the unobservable variable θ given by

$$\theta \sim N\left(0, \sigma_\theta^2\right)$$

Agents (indexed by j) observe a private noisy signal of θ given by

$$z(j) = \theta + \varepsilon(j) : \varepsilon(j) \sim N\left(0, \sigma_\varepsilon^2\right) \forall j$$

That is, all agents receive an equally precise signal of θ but agent j only observes his own signal $z(j)$. Define

$$\begin{aligned} \theta^{(k)} &\equiv \int E\left[\theta^{(k-1)} \mid z(j)\right] dj \\ \theta^{(0)} &\equiv \theta \end{aligned}$$

a) Find an expression for $\theta^{(k)}$. What is the limit as $k \to \infty$? Discuss the role played by the assumption of rational expectations in the sense of "model consistent expectations" in deriving your expression for $\theta^{(k)}$.

b) Consider the set up above, but where the signal is instead given by

$$z(j) = \theta + \delta \quad \forall j : \delta \sim N\left(0, \sigma_\delta^2\right) \forall j$$

Find a new expression for $\theta^{(k)}$. Interpret.

c) Consider the model of Morris and Shin (AER 2002). Utility of agent $i \in (0,1)$ is given by

$$U_i = -(1-r)(a_i - \theta)^2 - r\left(L_i - \overline{L}\right)$$

where a_i is the action taken by agent i and

$$L_i = \int (a_j - a_i)^2 \, dj$$

and

$$\overline{L} = \int L_j dj$$

Agents observe two signals of θ. The public signal y

$$\begin{aligned} y &= \theta + \eta \\ \eta &\sim N(0, \sigma_\eta^2) \end{aligned}$$

and the private signal x_i

$$\begin{aligned} x_i &= \theta + \varepsilon_i \\ \varepsilon_i &\sim N(0, \sigma_\varepsilon^2) \forall i \end{aligned}$$

The first order condition for expected utility maximization is given by

$$a_i = (1-r) E\left[\theta \mid x_i, y\right] + r E\left[\int a_j dj \mid x_i, y\right]$$

where $\int a_j dj$ ($\equiv \overline{a}$) is the average action across agents. Find κ in the optimal linear reaction function of agent i

$$a_i = \kappa x_i + (1-\kappa) y$$

Solve for equilibrium average action $\overline{a}$ as a function of θ and η.

d) Discuss how the equilibrium action depends on r.

e) Find the average action $\overline{a}$ as a function of higher order expectations about θ.

Question 4: Miscellaneous

This question allows for more heuristic answers than the previous ones, though you are still encouraged to use some algebra if you think that would make your answers more accurate/consice.

***On the impossibility of informationally efficient markets* by Grossman and Stiglitz (AER 1980).**

a) Explain intuitively Grossman and Stiglitz's result regarding the impossibility of informationally efficient markets when information gathering is costly. Carefully define all terminology used.

b) Explain intuitively the bounds on the costs of information that guarantees that some agents will and some agents will not buy the signal, i.e. the bounds that guarantees an interior solution.

Bounded rationality and learning.

c) Describe intuitively decreasing and constant gain learning. Give examples of when it is suitable to assume that agents use each learning technology.

d) Describe what the policy makers in the paper *The Conquest of U.S. Inflation: Learning and Robustness to Model Uncertainty* by Cogley and Sargent RED (2005) are learning about.

e) Describe how probabilities and expected losses of individual sub-models relates to the optimal policy in a Bayesian Robustness setting.