# THE INFORMATION REVEALED BY MARKET PRICES

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In this course we are studying models in which agents do not have full information about the state of the economy. However, most economists are quite content with assuming that agents know the state of the economy with certainty. The justifications for this assumption falls broadly into two categories: (i) Information imperfections do not matter, so we do not loose anything by abstracting from them, or, (ii) The assumption is meant literally. To make the first argument, one of course first have to solve imperfect information models and compare their predictions to those of full information models. To motivate the literal interpretation, it is common to assume that the state is an invertible function of prices. That is, markets convey all available information about the state through prices. In these notes we will first present an argument due to Grossman (1975) that without "noise", the market price may provide a sufficient statistic for agents to predict the state of the economy. Ironically, when prices are informationally efficient, i.e. summarizes perfectly the information available to any agent, the private incentive to be informed disappears. That is, in equilibrium, traders will find private information to be of no use.

When market prices are noisy, perhaps due to random supply, the incentive for individual agents to acquire costly information returns. However, as shown by Grossman and Stiglitz (1980), when information is costly to acquire, prices cannot perfectly aggregate the information in the economy perfectly, since if it did, no one would be willing to pay for the costly information if the same information is revealed by prices for free. This is the well-known Grossman-Stiglitz paradox.

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## 1. The price as a sufficient statistic

Grossman (1975) argues in a simple setting that the price is a sufficient statistic for predicting the pay-off of an asset and that the price can aggregates all the information available to any trader in the economy in an efficient way. The set up is as follows.

1.1. **Model Set Up.** There is a $n$ traders index by $j$ who divide their wealth between a risky asset with price $p$ and pay-off $\theta$ and a risk free asset with return that can be normalized to zero. The pay off $\theta$ on the risky asset is normally distributed

$$\theta \sim N\left(0, \sigma_\theta^2\right) \tag{1.1}$$

Initial wealth $w_0^j$ has to be divided between the risky asset $x$ and the safe asset $m$

$$w_j^0 = px_j + m_j \tag{1.2}$$

The terminal wealth of trader $j$ is then given by

$$w_j = \theta x_j + \underbrace{\left[w_j^0 - x_j p\right]}_{\text{wealth allocated to safe asset}} \tag{1.3}$$

Trader $j$ chooses his holdings $x_j$ of the risky asset in order to maximize

$$V\left(w_j\right) = E\left[-e^{-\gamma w_j} \mid \Omega^j\right] \tag{1.4}$$

and $\Omega^j$ is the information set of trader $j$.

1.2. **Optimal portfolios.** If traders' signals about $\theta$ are normally distributed, expected wealth will also be conditionally normal. We can then use the following trick to solve for optimal portfolios. For any variable

$$z \sim N(\mu_z, \sigma_z^2) \tag{1.5}$$

the expectation of the exponential $e^z$ is given by

$$E\left[e^{z}\right]=e^{\left(\mu_{z}-\frac{1}{2}\sigma_{z}^{2}\right)} \tag{1.6}$$

We can use this fact to evaluate the conditional expected utility

$$\begin{aligned} E\left[-e^{-\gamma w(j)} \mid \Omega^{j}\right] &= E\left[-e^{-\gamma\left(\theta x_{j}-\left[w_{j}^{0}-x_{j} p\right]\right)} \mid \Omega^{j}\right] && (1.7)\\ &= -\exp\left(-\gamma\left(x_{j} E\left[\theta \mid \Omega^{j}\right]-x_{j}\left[p-w_{j}^{0}\right]-\frac{\gamma}{2} Var\left(w_{j} \mid \Omega^{j}\right)\right)\right) && (1.8)\\ &= -\exp\left(-\gamma\left(x_{j} E\left[\theta \mid \Omega^{j}\right]-x_{j}\left[p-w_{j}^{0}\right]-\frac{\gamma}{2} x_{j}^{2} \sigma^{2}\right)\right) && (1.9) \end{aligned}$$

where $\sigma^2$ is the conditional variance of the pay off of the risky asset, i.e.

$$\sigma^{2} \equiv E\left(\theta-E\left[\theta \mid \Omega^{j}\right]\right)^{2} \tag{1.10}$$

To find the optimal portfolio, differentiate expected utility w.r.t. $x_j$

$$\frac{\partial E\left(w_{j}\right)}{\partial x_{j}}=-\gamma\left[E\left[\theta \mid \Omega^{j}\right]-p\right]+\gamma^{2} x_{j} \sigma^{2} \tag{1.11}$$

and set equal to zero and solve for trader $j$'s demand

$$x_{j}=\frac{\left(E\left[\theta \mid \Omega^{j}\right]-p\right)}{\gamma \sigma^{2}} \tag{1.12}$$

1.3. **Equilibrium price.** So far, this is a special case of the models studied in Admati (1985) except for the assumption that there is a finite number of traders. Another difference is that in Grossman (1975) supply of the asset is fixed at $\overline{x}$. Equating net demand and supply

$$\sum_{j=1}^{n} x_{j}=\overline{x} \tag{1.13}$$

yields

$$\frac{\sum_{j=1}^{n}\left(E\left[\theta \mid \Omega^{j}\right]-p\right)}{\gamma \sigma^{2}}=\overline{x} \tag{1.14}$$

the equilibrium price

$$p = n^{-1} \sum_{j=1}^{n} E\left[\theta \mid \Omega^j\right] - n^{-1}\gamma\sigma^2\overline{x} \tag{1.15}$$

1.4. **Traders' Information Sets.** The information set of trader $j$ is given by

$$\Omega^j \equiv \{z_j, p\}$$

where

$$z_j = \theta + \eta_j : \eta_j \sim N\left(0, \sigma_\eta^2\right) \ \forall \ \ j \tag{1.16}$$

so that all traders are ex ante symmetric.

1.5. **Price and signals.** We know that in a linear model with Gaussian shock, conditional expectations are linear functions of the signals so that

$$E\left[\theta \mid \Omega^j\right] = g_z z_j + g_p p \tag{1.17}$$

for some coefficients $g_z$ and $g_p$. Plugging this expression into the price function

$$p = n^{-1} \sum_{j=1}^{n} g_z z_j + g_p p - n^{-1}\gamma\sigma^2\overline{x}$$

and solving for $p$ gives

$$p = n^{-1} \sum_{j=1}^{n} \frac{g_z}{1-g_p} z_j - n^{-1}\frac{\gamma\sigma^2}{1-g_p}\overline{x} \tag{1.18}$$

Under rational expectations, $\frac{\gamma\sigma^2}{ng_p}\overline{x}$ is simply a known constant. Since traders are ex ante symmetric, i..e there are no $j$ subscripts on $g_z$ and $g_p$, we can rewrite the equilibrium price as

$$p = \frac{g_z}{1-g_p}\overline{z} - n^{-1}\frac{\gamma\sigma^2}{1-g_p}\overline{x} \tag{1.19}$$

where

$$\overline{z} \equiv \frac{1}{n}\sum z_j$$

In a rational expectations equilibria, traders are assumed to know the form of the price function as well as the equilibrium parameters. An individual trader can thus back out the average signal from the price by inverting the price function

$$\overline{z} = \frac{1-g_p}{g_z}p + \frac{\gamma\sigma^2}{g_z}\overline{x}$$

Grossman (1975) uses this expression together with Bayes Rule and a formal definition of a sufficient statistic to prove that in equilibrium, traders will disregard their private signal $z_j$ and only condition on the backed out measure of the average signal $\overline{z}$. The same argument can be made more intuitively (I think) by considering the filtering problem of a fictitious trader who can observe the entire vector of private signals and use that to form an expectation about $\theta$.

Let $\mathbf{z}$ denote the vector of all the signal in the economy

$$\mathbf{z} \equiv \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \tag{1.20}$$

The conditional expectation of $\theta$ conditional on $\mathbf{z}$ is then given by

$$\begin{aligned} E(\theta \mid z) &= \mathbf{g}'\mathbf{z} \\ &= \sum_{j=1}^{n} g_j z_j \\ &= cov(\theta, z)\left[var(z)\right]^{-1}\mathbf{z} \\ &= \begin{bmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \cdots & \sigma_\theta^2 \end{bmatrix}\left[\sigma_\theta^2 \times \mathbf{1}_{n\times n} + \sigma_\varepsilon^2 \times I_n\right]^{-1}\mathbf{z} \end{aligned}$$

By the symmetry of signals $z_j$, the fictitious agent will put equal weight on all the signal in the vector $\mathbf{z}$ so that

$$g_j = g \quad \forall j \tag{1.21}$$

We can thus write

$$
\begin{aligned}
E(\theta \mid z) &= \mathbf{g}'\mathbf{z} && (1.22)\\
&= g\sum_{j=1}^{n} z_j && (1.23)\\
&= ng\overline{z} && (1.24)
\end{aligned}
$$

That is, the conditional expectation of an agent that can observe all the signals in the economy coincides with the best prediction based only on the average signal. The average signal is thus optimally summarizes all the information in the economy and a trader who can observe the price $p$ will disregard his private signal $z_j$ in equilibrium. Note that this result do not depend on the number of agents being "large" and the noise terms in the individual signals do not have to sum to zero.

While markets are "informationally efficient" with a finite number of traders, Hellwig (1980) points out that the traders in the model of Grossman (1975) are somewhat schizophrenic in that they understand that the price is affected by their own demand, though do not act strategically to take advantage of this.

## 2. The Impossibility of Informationally Efficient Markets

Grossman and Stiglitz (1980) presents an elegant argument why markets cannot perfectly reveal all relevant information if information gathering is costly. It is a simple model of a single period asset market with traders with Constant Absolute Risk Aversion (CARA) preferences.

### 2.1. **The Model.**

There are two assets: A safe asset yielding return $R = 1$ and a risky asset yielding return $u$

$$u = \theta + \varepsilon \tag{2.1}$$

where $\theta$ is observable at cost $c$ and both $\theta$ and $\varepsilon$ are random variables with the following properties:

$$\begin{aligned} Eu &= \mu_u \\ E\varepsilon &= 0 \\ E\theta\varepsilon &= 0 \\ Var\left(u \mid \theta\right) &= Var\left(\varepsilon\right) \equiv \sigma_\varepsilon^2 > 0 \end{aligned}$$

There are two types of (ex ante identical) agents: Those who observe $\theta$ (informed traders) and those who don't (uninformed traders). That is, ex ante identical agents choose whether or not to pay the cost $c$ to observe the signal $\theta$.

Trader $j$'s (indirect) utility is given by

$$V\left(w_j\right) = E\left[-e^{-\gamma w_j} \mid \Omega^j\right] \tag{2.2}$$

where $\gamma$ is the coefficient of absolute risk aversion and $w_j$ is next period wealth. As we know, trader $j$'s optimal demand is then given by

$$x_j = \frac{\left[E\left(u \mid \Omega^j\right) - p\right]}{\gamma E\left[u - E\left(u \mid \Omega^j\right)\right]^2}$$

This expression for the demand is the same for both informed and uninformed agents. The difference will come from the information sets that these expectations are conditioned on. Informed traders condition on $\theta$ and will make conditional expectation errors of $u$ equal to $\varepsilon$ with variance $\sigma_\varepsilon^2$

$$x_I = \frac{E\left(u \mid \theta\right) - p}{\gamma E\left[u - E\left(u \mid \theta\right)\right]^2} \tag{2.3}$$

$$= \frac{E\left(u \mid \theta\right) - p}{\gamma \sigma_\varepsilon^2} \tag{2.4}$$

Uninformed traders will condition on the price P and make conditional errors with variance $\sigma^2_{u|p}$

$$x_U = \frac{E(u \mid p) - p}{\gamma E\left[u - E(u \mid p)\right]^2} \tag{2.5}$$

$$= \frac{E(u \mid p) - p}{\gamma \sigma^2_{u|p}} \tag{2.6}$$

Equilibrium price (for a given proportion $\lambda$ of informed traders and given conditional variances) can be found by equating aggregate demand and (exogenous) supply $x \sim N\left(\mu_x, \sigma_x^2\right)$

$$\lambda x_I + (1-\lambda) x_U = x \tag{2.7}$$

$$\lambda \frac{E(u \mid \theta) - p}{\gamma \sigma^2_{u|\theta}} + (1-\lambda) \frac{E(u \mid p) - p}{\gamma \sigma^2_{u|p}} = x \tag{2.8}$$

$$\Longleftrightarrow$$

$$\lambda \frac{p}{\gamma \sigma^2_{u|\theta}} + (1-\lambda) \frac{p}{\gamma \sigma^2_{u|p}} = -x + \lambda \frac{E(u \mid \theta)}{\gamma \sigma^2_{u|\theta}} + (1-\lambda) \frac{E(u \mid p)}{\gamma \sigma^2_{u|p}} \tag{2.9}$$

$$\Longleftrightarrow$$

$$p = \left[\left(\frac{\lambda}{\gamma \sigma^2_{u|\theta}} + \frac{(1-\lambda)}{\gamma \sigma^2_{u|p}}\right)\right]^{-1} \times \left[\lambda \frac{E(u \mid \theta)}{\gamma \sigma^2_{u|\theta}} + (1-\lambda) \frac{E(u \mid p)}{\gamma \sigma^2_{u|p}} - x\right] \tag{2.10}$$

In the case of all traders being informed, i.e. $\lambda = 1$, this simplifies to

$$p_{\lambda=1} = \left[E(u \mid \theta) - \gamma \sigma^2_{u|\theta} x\right] \tag{2.11}$$

$$= \theta - \gamma \sigma^2_{\varepsilon} x \tag{2.12}$$

That is, prices increase in expected return but decreases in risk aversion $\gamma$, conditional variance of the return $\sigma_\varepsilon^2$ and the return on the alternative investment (the safe asset).

$$\begin{aligned} p_{\lambda=0} &= \left[E(u \mid p) - \gamma\sigma^2_{u|p(\lambda=0)}x\right] && (2.13) \\ &= E(u) - \gamma\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)x && (2.14) \end{aligned}$$

where we used that the price is uninformative about $u$ when nobody buys the signal.

2.2. **Understanding when it pays to switch strategy.** Looking at two limit cases can help intuition for the decision of agents and when an (interior) equilibrium exists. First, note that the decision to buy the signal or not is made ex ante, so below we will have to replace random variables with their expected values.

2.2.1. *Case 1: Everybody is informed.* First, consider an agent in a world where everybody else has chosen to buy the signal $\theta$ and paying the cost $c$. The benefit of the decrease in conditional return variance from observing the signal instead of only observing the price (which can be observed for free) must then be larger than the cost $c$ in terms of expected utility. Formally, it will pay to switch strategy if the inequality holds

$$\begin{aligned} EV\left(w_j \mid \theta\right) &= -\exp\left(-\gamma\left[w_j^0 - c + E\left(x_{I,\lambda=1}\left[\theta - p_{\lambda=1}\right]\right) - \frac{\gamma}{2}E\left(x^2_{I,\lambda=1}\sigma_\varepsilon^2\right)\right]\right) && (2.15) \\ &< -\exp\left(-\gamma\left[w_j^0 + E\left(x_{U,\lambda=1}\left[E\left(u \mid p\right) - p_{\lambda=1}\right]\right) - \frac{\gamma}{2}E\left(x^2_{U,\lambda=1}\sigma^2_{u|p(\lambda=1)}\right)\right]\right) \end{aligned}$$

The first line is the expected utility of being informed when all traders are informed and the second line is the expected utility of being the only uninformed agent. We can simplify to get

$$\begin{aligned} c &> E\left[x_{I,\lambda=1}\left(\left[E\left(u\right) - p_{\lambda=1}\right] - \frac{\gamma}{2}x_{I,\lambda=1}\sigma_\varepsilon^2\right)\right] && (2.16) \\ &\quad -E\left[x_{U,\lambda=1}\left(\left[E\left(u\right) - p_{\lambda=1}\right] - \frac{\gamma}{2}x_{U,\lambda=1}\sigma^2_{u|p(\lambda=1)}\right)\right] \end{aligned}$$

where we used that the ex ante, the unconditional and conditional return expectation are the same (that is, the choice to buy the signal or not is made before the signal is observed). Obviously, for bounded conditional variances, there exists a $c$ large enough to make the inequality (2.16) hold. However, to check for the existence of an interior equilibrium we want to find an interval over which $c$ is large enough to make it profitable to not buy information when everybody else is doing so, but small enough to make it worthwhile when no one else is buying information. To do so we need to look at the opposite limit.

2.2.2. *Case 2: Everybody is uninformed.* If everybody is uninformed, we need that the expected utility of switching to being informed outweighs the cost of the signal so that the inequality

$$\begin{aligned} EV\left(w_j \mid \theta\right) &= -\exp\left(-\gamma\left[w_j^0 - c + E\left(x_{I,\lambda=0}\left[\theta - p_{\lambda=0}\right]\right) - \frac{\gamma}{2}x_{I,\lambda=0}^2\sigma_\varepsilon^2\right]\right) \quad (2.17) \\ &> -\exp\left(-\gamma\left[w_j^0 + E\left(x_{U,\lambda=0}\left[E\left(u \mid p\right) - p_{\lambda=0}\right]\right) - \frac{\gamma}{2}x_{U,\lambda=0}^2\sigma_{u|p(\lambda=1)}^2\right]\right) \end{aligned}$$

must hold for some c for there to exist an interior solution. Again, rearranging and simplifying yields

$$\begin{aligned} c &< E\left[x_{I,\lambda=0}\left(\left[E\left(u\right) - p_{\lambda=0}\right] - \frac{\gamma}{2}x_{I,\lambda=0}\sigma_\varepsilon^2\right)\right] \quad (2.18) \\ &\quad - E\left[x_{U,\lambda=0}\left(\left[E\left(u\right) - p_{\lambda=0}\right] - \frac{\gamma}{2}x_{U,\lambda=0}\sigma_{u|p(\lambda=0)}^2\right)\right] \end{aligned}$$

That is, there must exists a cost smaller than the benefit of buying the signal when nobody else does so. For an interior solution to exists, there must be a cost small enough for the marginal trader to deviate from the strategy of everybody else at both limit points, i.e. both to buy the signal when nobody else does as well as not buy the signal when everybody else does.

We can think of the right hand side of (2.16) as an lower bound $\underline{c}$ on the interval and the right hand side of (2.18) as the upper bound $\overline{c}$ on the interval of costs that will yield interior solutions. (If $\overline{c} < \underline{c}$ no interior solution exists.)

We could verify the inequalities above numerically by computing the conditional variances $\sigma^2_{u|p(\lambda=0)}$ and $\sigma^2_{u|p(\lambda=1)}$. This can be done by exploiting the following relationships

$$E\left(p_{\lambda=1}\right) = E(u) - \gamma\sigma^2_\varepsilon E\left(x\right) \tag{2.19}$$

$$E\left(p_{\lambda=0}\right) = E(u) - \gamma\left(\sigma^2_\theta + \sigma^2_\varepsilon\right) E\left(x\right) \tag{2.20}$$

$$E\left(x_{I,\lambda=1}\right) = \frac{E\left(u\right)}{\gamma\sigma^2_\varepsilon} - \left(\frac{E(u)}{\gamma\sigma^2_\varepsilon} - \frac{\gamma\sigma^2_\varepsilon}{\gamma\sigma^2_\varepsilon}E\left(x\right)\right) = E\left(x\right) \tag{2.21}$$

$$E\left(x_{U,\lambda=1}\right) = \frac{E\left(u\right)}{\gamma\sigma^2_{u|p(\lambda=1)}} - \left(\frac{E(u)}{\gamma\sigma^2_{u|p(\lambda=1)}} - \frac{\gamma\sigma^2_\varepsilon}{\gamma\sigma^2_{u|p(\lambda=1)}}E\left(x\right)\right) = \frac{\sigma^2_\varepsilon}{\sigma^2_{u|p(\lambda=1)}}E\left(x\right) \tag{2.22}$$

$$E\left(x_{I,\lambda=0}\right) = \frac{E\left(u\right)}{\gamma\sigma^2_\varepsilon} - \left(\frac{E(u)}{\gamma\sigma^2_\varepsilon} - \frac{\gamma\sigma^2_{u|p(\lambda=0)}}{\gamma\sigma^2_\varepsilon}E\left(x\right)\right) = \frac{\sigma^2_{u|p(\lambda=0)}}{\sigma^2_\varepsilon}E\left(x\right) \tag{2.23}$$

$$E\left(x_{U,\lambda=0}\right) = E\left(x\right) \tag{2.24}$$

together with the fact that the conditional variance of joint normally distributed variables $Y$ and $X$ are given by

$$E\left[E\left(Y \mid X\right) - Y\right]^2 = E\left(YY'\right) - E\left(YX'\right)E\left(XX\right)^{-1}E(XY') \tag{2.25}$$

Using this formula and the expression for the price, the variance of returns conditional on $p_{\lambda=1}$ is then given by

$$\begin{aligned} E\left[E\left(u \mid p_{\lambda=1}\right) - u\right]^2 &= E\left(u^2\right) - E\left(up_{\lambda=1}\right)E\left(p_{\lambda=1}p_{\lambda=1}\right)^{-1}E(p_{\lambda=1}u) && (2.26)\\ &= \left(\sigma^2_\theta + \sigma^2_\varepsilon\right) - \sigma^2_\theta\left(\sigma^2_\theta + \left(\gamma\sigma^2_\varepsilon\right)^2\sigma^2_x\right)^{-1}\sigma^2_\theta && (2.27)\\ &= \left(\sigma^2_\theta + \sigma^2_\varepsilon\right) - \frac{\left(\sigma^2_\theta\right)^2}{\sigma^2_\theta + \left(\gamma\sigma^2_\varepsilon\right)^2\sigma^2_x} && (2.28)\end{aligned}$$

and the variance of returns conditional on $p_{\lambda=0}$ are simply

$$E\left[E\left(u \mid p_{\lambda=0}\right)-u\right]^2=\left(\sigma_\theta^2+\sigma_u^2\right) \tag{2.29}$$

since $E\left(up_{\lambda=0}\right)=0$.

2.3. **Grossman and Stiglitz impossibility argument.** It is now hopefully straightforward to understand the main argument of Grossman and Stiglitz's paper: Prices cannot be fully revealing if there is a positive cost $c$ of observing the signal $\theta$. To see why, consider the equilibrium conditional that expected utility of being informed must be the same as the expected utility of being uninformed

$$\frac{-\exp\left(-\gamma\left[w_j^0-c+x_I\left[E\left(u \mid \theta\right)-p\right]-\frac{\gamma}{2}x_I\sigma_\varepsilon^2\right]\right)}{-\exp\left(-\gamma\left[w_j^0+x_U\left[E\left(u \mid p\right)-p\right]-\frac{\gamma}{2}x_U\sigma_{u|p}^2\right]\right)}=1 \tag{2.30}$$

If prices are perfectly revealing, allocations and conditional expectations and variances will be identical across informed and uninformed agents. That is

$$x_I = x_U \tag{2.31}$$

$$E\left(u \mid \theta\right) = E\left(u \mid p\right) \tag{2.32}$$

$$\sigma_\varepsilon^2 = \sigma_{u|p}^2 \tag{2.33}$$

must hold if prices are perfectly revealing of the information in the signal $\theta$. But since there is no benefit from observing the signal $\theta$ no one would be willing to pay a cost to do so, and we can only have a perfectly revealing equilibrium if information is for free, that is $c=0$. Another way of seeing this more clearly (at least algebraically) is to define $K$

$$K \equiv w_j^0+x_I\left[E\left(u \mid \theta\right)-p\right]-\frac{\gamma}{2}x_I^2\sigma_\varepsilon^2 \tag{2.34}$$

$$= w_j^0+x_U\left[E\left(u \mid p\right)-p\right]-\frac{\gamma}{2}x_U^2\sigma_{u|p}^2 \tag{2.35}$$

so that the equilibrium condition (2.30) under fully revealing prices can be written as

$$\frac{-K+c}{-K} = 1 \tag{2.36}$$

which obviously cannot hold for any $c > 0$.

## 3. Mechanical conditions to check if a system is perfectly revealing

The following results from Baxter, Graham and Wright (JEDC 2010) can be useful:

Consider the linear system of the form

$$\underset{(n\times 1)}{X_t} = AX_{t-1} + B \underset{(m\times 1)}{\mathbf{w}_t} \; : \mathbf{w}_t \sim N(0,\mathbf{I}) \tag{3.1}$$

$$\underset{(s\times 1)}{Z_t} = DX_t \tag{3.2}$$

**Definition 1.** *The information set* $I_t \equiv \{Z_t, Z_{t-1}, ..., Z_0\}$ *is said to be instantaneously invertible if* $rank(D) = n$ *so that* $D$ *is invertible.*

If the information set $I_t$ is instantaneously invertible, the last observation is sufficient to extract a perfect estimate of the state.

**Definition 2.** *The information set* $I_t \equiv \{Z_t, Z_{t-1}, ..., Z_0\}$ *is said to be asymptotically invertible* $iff$

(1) $rank(B) = s$
(2) $DB'$ *is invertible*
(3) $\left|eig\left[\left(I - B\left(DB\right)^{-1}D\right)\right]A\right| < \mathbf{1}$

If these condition are satisfied, the posterior state uncertainty

$$P_{t|t} \equiv E\left(X_t - E\left[X_t \mid I_t\right]\right)\left(X_t - E\left[X_t \mid I_t\right]\right)' \tag{3.3}$$

tend to zero as $t$ increases. That is, $\lim_{t\to\infty} P_{t|t} = \mathbf{0}$. If conditions (1) -(3) are not satisfied, the state $X_t$ can never be perfectly recovered from $Z^t$so that $\lim_{t\to\infty} P_{t|t} \neq \mathbf{0}$

3.1. **A rule-of-thumb.** A rule of thumb which in most cases gives the same answer as the formal condition above is to count shocks, including both structural innovations and measurement errors. If this is a larger number than the number of observable variables, the state will not be perfectly revealed.

3.2. **An easy check for models that are difficult to solve.** If the imperfect information solution is hard to compute and you want to check whether equilibrium outcomes will reveal the state before you go through the trouble of solving the model, there is a way: if the model is easy to solve under perfect information, you can check if the state is an invertible function of equilibrium outcomes in the perfect information solution to the model. The reason this works is that if equilibrium outcomes reveals the state perfectly, the equilibrium in question must be the perfect information equilibrium. So this strategy allow you to check if the model you have in mind will support information imperfections in equilibrium without solving for an imperfect information equilibrium. But the rule-of-thumb is of course even easier.

## References

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