Empirical Implications of Information Imperfections

March 14, 2016
The Plan:

- Noisy signals and business cycles
- Speculation and asset prices
Sentiments, Animal Spirits and Undue Optimism/Pessimism
An Old Idea

“The varying expectations of business men ... and nothing else, constitute the immediate cause and direct causes or antecedents of industrial fluctuations.”

A.C. Pigou
“Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits, a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.”

J.M. Keynes
Many terms, few formal definitions:

Undue Optimism, Animal spirits, Sentiments...what does it all mean?

Today we will consider undue optimism or sentiment defined as optimistic/pessimistic expectations about future productivity that are not motivated by true fundamentals

- Expectations can still be rational

Question: Can we quantify the importance of undue optimism empirically?
A Theory of Demand Shocks, Lorenzoni (AER 2009)

Do you remember...?

Basic set up

- Lucas (1972) style island model with informationally isolated islands
- Standard New-Keynesian model with sticky prices and monopolistic competition
- Aggregate productivity follows a random walk
- Optimal consumption determined by permanent income hypothesis

A public noisy signal will introduce undue optimism and pessimism
A public signal

In addition to the island specific source of information, all islands also observe a public signal about aggregate productivity

\[ s_t = x_t + e_t \]

Public signals improve average estimates, but at times must induce agents to respond to "false alarms" (i.e. \( \Rightarrow \) "demand shocks")
The effects of noise shocks

Figure 1. Impulse responses of output, employment, inflation, and the interest rate

A. Responses to the Three Shocks
Quantifying the importance of noise shocks empirically
Quantifying the importance of noise shocks empirically

How can we quantify the role of news shocks empirically?

Questions:

- Do we have direct empirical measures of sentiments or undue optimism/pessimism?
- What type of identification strategies can be used to quantify the effect of noise shocks?

Potential strategies:

- Long-run restrictions a la Blanchard and Quah (1989)
- Using structural models, e.g. Barsky and Sims, AER 2012 and Blanchard, L’Huillier and Lorenzoni, AER 2014.
Empirical measures of sentiments, animal spirits and/or undue optimism/pessimism

Barksy and Sims (2014) argue that survey data on Consumer Confidence may be helpful.

Main Survey Question:
*Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?*

Other questions ask about personal economic conditions.
Consumer Confidence and Recessions
Direction of causation

Clearly, the confidence index dips during recessions
But:
   ▶ Is confidence low because of the recessions?
   ▶ Is there a recessions because confidence is low?

There are some interesting challenges involved in identifying the direction of causality

To have some hope to use the Consumer Index to identify the effect of undue optimism there must be some independent variation in the index.
Identifying the Effect of Noise Shocks using SVARs

SVARs will not work:

- If there is a response to a pure noise shock, then this is a “mistake” from the perspective of the agents
- We cannot identify a mistake made by rational agents from variables that the agents also can observe

Remember the projection theorem:

\[ E[(X - \beta Y)Y'] = 0 \]

We cannot use anything in \( Y \) to predict \( X - \hat{X} \)
Identifying Noise and News Shocks using DSGEs

DSGEs could potentially work:

- Use ML to estimate deep parameters and quantify the contribution to overall variance made by noise shocks
- Kalman smoother can be used to back out time series of noise shocks

This strategy is pursued by Blanchard, L’Hullier and Lorenzoni (AER 2013)
News, Noise and Fluctuations: An empirical Investigation (BLL 2013)

Sets up a simple representative agent version of the model Lorenzoni (2009)

- Permanent and transitory productivity shocks
- Noisy signal about permanent component
- Estimate model using standard macro time series
## Prior and posterior estimates

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<th>Posterior</th>
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<th>Distribution</th>
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### Shock processes

Neutral technology and noise

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Investment-specific

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Impulse Response Functions (Quantities)

Figure 5. Impulse Responses, Bayesian DSGE, Quantities
Impulse Response Functions (Prices)

Figure 6. Impulse Responses, Bayesian DSGE, Prices
## Variance decompositions

### Table 6—Variance Decomposition

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Estimating the agents’ mistakes using the Kalman Smoother

The standard filter gives an optimal \textit{real time} estimate of the latent state

- Sometimes we are interested in the best estimate given the complete sample, i.e. $X_{t|T}$

\[
X_{t|T} = E \left[ X_t \mid Z^T, X_{0|0} \right]
\]

The \textit{Kalman smoother} can be used to find $X_{t|T}$

Because we use more information than what is available to agents in \textit{real time} we can quantify their mistakes
The Kalman Smoother: Implementation

Run filter forward, then backward.

\[ X_{t|T} = X_{t|t} + J_{t-1} (X_{t+1|T} - X_{t+1|t}) \]

where

\[ J_t = P_{t|t} A' P^{-1}_{t+1|t} \]

The covariances of the smoothed state estimation errors can be computed as

\[ P_{t|T} = P_{t|t} + J_t (P_{t+1|T} - P_{t+1|t}) J_t' \]

(for more details, see Hamilton 1994).
Estimated true and expected productivity

Figure 3. Smoothed Estimates of the Permanent Component of Productivity, of Long-Run Productivity, and of Consumers’ Real Time Expectations
Estimated history of exogenous shocks

**Figure 4. Smoothed Estimates of the Shocks**
Speculation, Information and Asset Prices
Speculation, Information and Asset Prices

“Markets are concerned with the ultimate health of economies and the like but they are equally, or more, concerned with what the likely judgments of other market participants in the short run are.”

Larry Summers, Lunch with the FT interview, 2009
Speculation, Information and Asset Prices

Heterogeneous information introduces a Keynesian “beauty contest” element to multi-period asset markets (e.g. Allen, Morris and Shin, RFS 2006)

- Prices are determined by agents that ”forecast the forecast of others” (Townsend JPE 1983)
- Agents will take speculative positions against what they perceive to be inaccurate “market expectations”

How can we empirically quantify speculation due to heterogeneous information?
Speculation, Risk Premia and Expectations in the Yield Curve (Barillas and Nimark 2014)

1. Propose an affine no-arbitrage model for asset pricing allowing for heterogeneously informed agents

2. Estimate a model using bond yields and individual responses from the Survey of Professional Forecasts

3. Decompose bond yields into short-rate expectations, risk premia and a statistically distinct speculative component
The Economy

The agents:
- Ex-ante identical and indexed by $j \in (0, 1)$
- Price takers
- Ex-post partly heterogeneous information sets $\Omega^j_t$
- Form rational (i.e. model consistent) expectations

The assets:
- A one-period risk free asset with return $r_t$
- Zero-coupon no-default bonds of maturities 2, 3, ..., $\bar{n}$ periods

The price of the risky assets will be determined by no-arbitrage conditions
No-arbitrage pricing of zero coupon bonds

**Common information**
No-arbitrage under common information implies that

\[ P_t^n = E \left[ M_{t+1} P_{t+1}^{n-1} \mid \Omega_t \right] \]

must hold for each maturity \( n \).

**Heterogeneous information**
No-arbitrage under heterogeneous information implies that

\[ P_t^n = E \left[ M_{t+1}^i P_{t+1}^{n-1} \mid \Omega_t^i \right] \]

must hold for each maturity \( n \) and each agent \( j \).
Higher order expectations and the price of a bond

Recursive forward substitution of no-arbitrage condition

\[ P_t^n = E \left[ M_{t+1}^i P_{t+1}^{n-1} \mid \Omega_t^i \right] \]

until the period of maturity gives

\[ P_t^n = E \left[ M_{t+1}^i E \left[ M_{t+2}^i \ldots E \left[ M_{t+n}^{i'} \mid \Omega_{t+n-1}^{i'} \right] \ldots \mid \Omega_{t+1}^i \right] \mid \Omega_t^i \right] \]

The price can thus be expressed as a function of higher order expectations about future SDFs.
Higher order expectations and the price of a bond

Taking logs and using that no-arbitrage condition also applies to the “average agent” gives

\[ p^n_t = \int E \left[ m^i_{t+1} \mid \Omega^i_t \right] dj \]

\[ + \int E \left[ \int E \left[ m^i_{t+2} \mid \Omega^i_{t+1} \right] di \mid \Omega^i_t \right] dj + \ldots \]

... + \int E \left[ \int E \left[ \ldots \int E \left[ m^{i'n}_{t+n} \mid \Omega^{i'n}_{t+n-1} \right] di' \ldots \right] di \mid \Omega^i_t \right] dj

+ constant

More convenient to derive the speculative component in a bond’s log price
Defining speculation

“Investors exhibit speculative behavior if the right to resell a stock makes them willing to pay more for it than they would pay if obliged to hold it forever”

Harrison and Kreps (1978)
The counterfactual consensus price

The “consensus price” \( \bar{p}_t^n \) is the price that would prevail if, by chance, agents’ higher order expectations coincided with their first order expectation.

\[
\bar{p}_t^n = \sum_{s=1}^{n} \int E \left[ m_{t+s}^i \mid \Omega_t^i \right] dj + \text{constant}
\]

Perceived agreement \( \Rightarrow \) no speculative motive
The Speculative Component

The speculative component $p_t^n - \bar{p}_t^n$ can be expressed as a sum of higher order prediction errors about future discount factors.

\[
    p_t^n - \bar{p}_t^n = - \int E \left[ m_{t+1}^j - m_{t+1}^j \mid \Omega_t^j \right] dj \\
    - \int E \left[ m_{t+2}^i - \int E \left[ m_{t+2}^i \mid \Omega_{t+1}^i \right] di \mid \Omega_t^j \right] dj - ... \\
    - \int E \left[ m_{t+n}^i - \int \int \int E \left[ m_{t+n}^i \mid \Omega_{t+n-1}^i \right] di' ... \right] di \mid \Omega_t^j \right] dj
\]

The speculative component is positive if individual agents think that other agents will underestimate future discount rates.
The Speculative Component and Public Information

**Proposition.** The speculative component $p^n_t - \bar{p}^n_t$ is orthogonal to public information in real time, i.e.

$$E \left( [p^n_t - \bar{p}^n_t] \omega_t \right) = 0 : \forall \omega_t \in \Omega_t$$

where $\Omega_t$ is the public information set at time $t$ defined as the intersection of agents' period $t$ information sets

$$\Omega_t \equiv \bigcap_{j \in (0,1)} \Omega^j_t.$$

**Proof.** The proof follows from the law of iterated expectations $E \left[ E \left[ X | \Omega \right] \Omega' \right] = E \left[ X | \Omega' \right]$ if $\Omega' \subseteq \Omega$ and that the definition of the public information set implies that $\Omega_t \subseteq \Omega^j_t$ for all $j$. 
Quantifying the importance of speculation in bond markets
The affine SDF approach to asset pricing

Affine no-arbitrage models impose weak restrictions that nest more structural (and restrictive) models.


Barillas and Nimark (2014) extend this modeling approach to a setting with privately informed agents

- Set up nests the standard full information affine Gaussian model
Affine No-arbitrage Term Structure Models

- Empirically very flexible
- Imposes weak restrictions (only no-arbitrage)
- Short rate and SDF affine functions of factors
The term structure model with heterogeneous information

The solved model is of the form

\[ X_{t+1} = \mu_X + FX_t + Cu_{t+1} \]
\[ p^n_t = A_n + B'_n X_t + \nu^n_t \]
The speculative component in the affine model

The consensus price $\bar{p}_t^n$ can be computed as

$$\bar{p}_t^n \equiv A_n + B'_n \bar{H} X_t + v_t^n$$

where

$$\begin{bmatrix}
    x_t \\
    x^{(1)}_t \\
    \vdots \\
    x^{(1)}_t \\
    x_t
  \end{bmatrix} = \bar{H} 
\begin{bmatrix}
    x_t \\
    x^{(1)}_t \\
    \vdots \\
    x^{(k)}_t \\
    x_t
  \end{bmatrix}$$

The speculative term is then simply given by

$$p_t^n - \bar{p}_t^n = \left( B'_n - B'_n \bar{H} \right) X_t$$
Estimating the parameters of the affine model
Estimation Method and Data

- Simulate posterior distribution using Metropolis-Hastings Algorithm
- Use quarterly data 1971:Q4 to 2011:Q4 on 1 to 10 year bond yields
- Use individual survey responses in Survey of Professional Forecasters
Cross-section of Survey Responses

Histogram of Survey Responses 2003:Q3 (1-quarter-ahead FFR forecasts)

Figure: Relative conditional yield variance with private signals
Using individual survey responses data in likelihood based estimation

Individual responses from the SPF are treated as representing the expectations of a trader drawn from the population of the model:

\[
E \left[ r_{t+1} \mid \Omega_t^j \right] \sim N \left( -A_1 - B_1 FHX_t, B_1 F \Sigma_j F' B_1' \right)
\]

where \( \Sigma_j \) is the cross-sectional covariance of expectations about the current state, i.e.

\[
\Sigma_j \equiv EH \left( X_t^j - X_t \right) \left( X_t^j - X_t \right)' H'
\]

We use one quarter ahead forecasts of the Federal Funds Rate and 10 year yields.
Risk-neutral dynamics and the informativeness of bond yields

The dispersion in the survey data restricts how informed agents can be about the latent factors.

Two sources of information:

1. Private signals
2. Bond yields

Too precise signals will imply a counterfactually degenerate distribution of expectations.
Risk-neutral dynamics and the informativeness of bond yields

Bonds are priced “as if” traders were risk neutral and \( r_t \) followed the risk neutral dynamics

- Factors with similar risk-neutral dynamics will have similar implication for the cross-section of yields and therefore be difficult to filter out from observing yields
- Factor with similar risk-neutral dynamics may still have very different implications for future yields

The risk-neutral dynamics encode a combination of actual (physical) dynamics and agents’ attitudes towards risk.
Risk-neutral persistence and the yield curve

Figure: The effect of a very persistent factor on the yield curve.
Figure: The effect of a moderately persistent factor on the yield curve.
Figure: The effect of a transitory factor on the yield curve.
The fit of the estimated model

Dynamics and unconditional moments of bond yields

- S.d. of maturity specific shocks are 50 basis points, comparable to pricing errors in full information model
- Unconditional yields are within a few basis points of sample means

Dispersion of survey responses

- S.d. of one-quarter-ahead forecast of risk-free short rate is 43 basis points (Data=40 b.p.)
- S.d. of one-quarter-ahead forecast of 10 year yield is 27 basis points (Data=40 b.p.)

All parameters appear to be well identified.
The quantitative importance of speculation
A three-way decomposition of bond prices

Risk premia, (first order) expectations about future short rates and the speculative term are linear functions of the state $X_t$

$$p_t^n = \underbrace{A_n^{rp} + B_n^{rp'} X_t}_{\text{classic risk premia}} + \underbrace{A_n^r + B_n^{r'} X_t}_{\text{short rate expectations}} + \underbrace{B_n' (I - H) X_t + v_t^n}_{\text{speculative term}}$$

We can use the Kalman smoother to get an estimate $E \left[ X^T \mid \bar{z}^T \right]$ conditional on the entire sample.
Historical decomposition

10-year Yield Decomposition

- Average Short Rate Expectations
- Speculative Term
- Risk Premia
- 10-year Yield

%
Heterogeneous vs full information

We can compare estimates of classical components of yield curve to those from the nested full information model of Joslin, Singleton and Zhu (2011) estimated on the same data.

- More of yield variation explained by (first order) expectations of future risk-free interest rates in heterogenous information model
- Risk premia in heterogeneous information model is less volatile than and negatively correlated with risk premia in full information model

It may thus be important to allow for heterogenous information even when primary interest is not in speculative dynamics
Heterogeneous vs full information

Average Short Rate Expectations for 10-year yield

Common Risk Premia for 10-year yield
How useful are the agent specific signals?

Agent specific signals $x_t^i$ are used to make more accurate predictions about future bond prices

- Without utility functions and portfolios, we cannot evaluate the usefulness of the trader specific signals in terms of welfare or profits
- But we can quantify the reduction in conditional yield variance due to private information relative to conditioning only on publicly observed bond yields

Agent specific signals increase $R^2$ by about 6 percentage points relative to conditioning only on bond yields (across all maturities)
Speculation, Information and the Term Structure

- We can exploit information in cross-sectional dispersion of survey responses to discipline the degree of information heterogeneity.
- Speculative dynamics are potentially quantitatively important and account for up to a percentage point of bond yields.
- Risk premia estimates in speculative model are less volatile and negatively correlated with risk premia from nested full information model.
General lessons:

- If agents are rational, we cannot use information available in real-time to all agents to estimate/quantify the effect of mistakes
  - Often makes regression based estimation infeasible
- Survey data can be used to discipline degree of information imperfections/heterogeneity
  - Restricts how informative equilibrium outcomes can be