Online Appendix to the paper

Speculation and the Bond Market: An Empirical No-arbitrage Framework

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Part I: Maturity specific shocks in affine and equilibrium models

This Appendix present a simple equilibrium model with heterogeneously informed agents and stochastic bond supply based on Nimark (2015). Here, we demonstrate that the affine no-arbitrage model with maturity specific shocks presented in the main text of the paper nests the equilibrium model as a special case and derive conditions for their equivalence.[1] The random supply shocks in the equilibrium model are shown to be formally equivalent to the maturity specific shocks in the affine model both under full and heterogeneous information. Since the algebra is a little simpler under full information, we first show that the equilibrium model with stochastic supply under full information is a restricted special case of a full information, affine no-arbitrage model with maturity specific shocks. It is then straightforward to verify the same equivalence between the two models when agents are heterogeneously informed.

1 Common objects in the affine and equilibrium model

We first define some objects and notation that is common to the both the equilibrium and affine model.

1.1 Agents

There is a continuum of agents indexed by $j \in (0,1)$ and agent j's information set is denoted Ω_t^j. Agents form rational expectations about future bond prices conditional on their information sets.

[1] The equilibrium model, which is arbitrage-free by construction, could then of course also be referred to as an "affine, no-arbitrage model". However, due to lack of a better terminology, we will refer to the equilibrium model as the "equilibrium model" and the model presented in the main paper as the "affine model".

1.2 The process for the short rate

The risk-free short rate r_t is an affine function of the factors x_t

$$r_t = \delta_0 + \delta_x' x_t \tag{1.1}$$

and the factors x_t follow the vector autoregressive process

$$x_{t+1} = \mu^{\mathbb{P}} = F^{\mathbb{P}} x_t + C u_{t+1} : u_{t+1} \sim N(0, I) \tag{1.2}$$

The vector u_{t+1} contains both the innovations to the factors x_{t+1} as well as the supply/maturity specific shocks that will be introduced below.

1.3 The state

Under full information, the state is simply the vector x_t which then is assumed to be perfectly observed by the agents. Under heterogeneous information, the state is the vector of higher order expectations X_t defined as

$$X_t \equiv \begin{bmatrix} x_t \\ x_t^{(1)} \\ \vdots \\ x_t^{(\bar{k})} \end{bmatrix} \tag{1.3}$$

where the average k order expectation $x_t^{(k)}$ is defined recursively as

$$x_t^{(k)} \equiv \int E\left[x_t^{(k-1)} \mid \Omega_t^j\right] dj. \tag{1.4}$$

The integer $\bar{k}$ is the maximum order of expectations considered. The state X_t follows a first order vector auto regression

$$X_{t+1} = \mu^X + \mathcal{F} X_t + \mathcal{C} u_{t+1} \tag{1.5}$$

where u_{t+1} is a vector containing all aggregate shocks, i.e. both innovations to the true factors x_t and the maturity specific/supply shocks (both defined below) since these affect agents (higher order) expectations about x_t.

1.4 The conjectured bond price equations

1.4.1 Full information

Under full information, the price of an n-period bond will be a function of the form

$$p_t^n = A_n + B_n' x_t + v_t^n \tag{1.6}$$

both in the equilibrium model and in the affine model.

1.4.2 Heterogeneous information

Under heterogeneous information, the price of an n-period bond will be a function of the form

$$p_t^n = A_n + B_n' X_t + v_t^n \tag{1.7}$$

both in the equilibrium model and in the affine model.

1.5 Bond supply/maturity specific shocks

In the equilibrium model, v_t^n is a shock to the supply of n-period bonds. In the affine models v_t^n is a maturity specific shock. In both models, the row vector V_n selects the appropriate shock from the vector u_t of aggregate shocks so that

$$v_t^n = V_n u_t \tag{1.8}$$

The supply/maturity specific shocks are collected in the vector v_t

$$v_t \equiv \begin{bmatrix} v_t^2 \\ \vdots \\ v_t^{\overline{n}} \end{bmatrix} \tag{1.9}$$

$$= V u_t \tag{1.10}$$

where $\overline{n}$ is the maximum maturity considered. Since there is no maturity specific shock to the short rate r_t, we define as $V_1 = \mathbf{0}$.

2 A simple equilibrium term structure model

Here we briefly describe the equilibrium model in Nimark (2015) which will be shown to be a restricted special case of the affine framework developed in the paper, both under full and heterogenous information.

Time is discrete and indexed by t. Each generation consists of a continuum of households with unit mass. Agent j invests one unit of wealth in period t on behalf of all households born in period t. In period $t+1$ agent j unwinds the position of the now old generation of households who then use the proceeds to consume. Agents are infinitely lived and perform the same service for the next generation of households.

2.0.1 Agent j's portfolio decision

There are two types of assets. A risk-free one period bond with (log) return r_t and risky zero-coupon bonds of maturities between 2 and $\overline{n}$. As in Singleton (1987) and Allen, Morris and Shin (2006), agents have a one-period investment horizon. Agent j chooses a vector of portfolio weights α_t^j in order to maximize the discounted expected log of wealth under management W_{t+1}^j in period $t+1$. That is, agent j solves the problem

$$\max_{\alpha_t^j} E\left[\log W_{t+1}^j \mid \Omega_t^j\right] \tag{2.1}$$

subject to

$$W_{t+1}^{j} = 1 + r_{p,t}^{j} \tag{2.2}$$

where Ω_t^j denotes agent j's information set and $r_{p,t}^j$ is the log return of the portfolio chosen by agent j in period t.

In the model presented below, equilibrium log returns of individual bonds are normally distributed. However, the log return on a portfolio of assets with individual log normal returns is not normally distributed. Following Campbell and Viceira (2002a, 2002b) we therefore use a second order Taylor expansion to approximate the log excess portfolio return as

$$r_{p,t}^{j} - r_t = \alpha_t^{j\prime}\left(\mathbf{p}_{t+1}^{-} - \mathbf{p}_t - \mathbf{r}_t\right) + \frac{1}{2}\alpha_t^{\prime j} diag\left[\Sigma_{p,t}^{j}\right] - \frac{1}{2}\alpha_t^{j\prime}\Sigma_{p,t}^{j}\alpha_t^{j} \tag{2.3}$$

where $\mathbf{p}_t$ is a vector of (log) bond prices for bonds of different maturities. The vector $\mathbf{p}_{t+1}^{-}$ contains the prices of the same bonds in the next period when they have one period less to maturity and $\mathbf{r}_t$ is a conformable vector in which each element is the risk-free rate r_t. The difference $p_{t+1}^{-}-\mathbf{p}_t-\mathbf{r}_t$ is thus a vector of excess returns on bonds of different maturities. The matrix $\Sigma_{p,t}^{j}$ is the covariance of log bond returns conditional on agent j's information set. Conditional returns are normally distributed, time invariant and with a common conditional covariance across all agents. We therefore suppress the subscripts and agent indices on the conditional return covariance matrix and write Σ_p instead of $\Sigma_{p,t}^{j}$ for all t and j. Current bond prices $\mathbf{p}_t$ and the current short rate r_t are assumed to be observed perfectly by all agents so that

$$\{\mathbf{p}_t, r_t\} \in \Omega_t^j \;\; \forall\, j \tag{2.4}$$

Maximizing the expected return (2.3) on wealth under management by agent j with respect to α_t^j then gives the optimal portfolio weights

$$\alpha_t^j = \Sigma_p^{-1}\left(E\left[\mathbf{p}_{t+1}^{-} \mid \Omega_t^j\right] - \mathbf{p}_t - \mathbf{r}_t\right) + \frac{1}{2}\Sigma_p^{-1} diag\left[\Sigma_p\right] \tag{2.5}$$

Since each agent j has one unit of wealth to invest, taking average across agents of the portfolio weights (2.5) yields the aggregate demand for bonds.

2.1 Bond supply

The vector of bond supply s_t is stochastic

$$s_t = \Sigma_p^{-1} v_t : v_t \sim N\left(0, VV'\right) \tag{2.6}$$

To simplify notation, the vector of supply shocks v_t are normalized by the inverse of the conditional variance of bond prices Σ_p^{-1}. The supply shocks v_t play a similar role here as the noise agents in Admati (1985). That is, they will prevent equilibrium prices from fully revealing the information held by other agents. While there may be some uncertainty about the total number of bonds outstanding, an economically more appealing interpretation of the supply shocks is in terms of effective supply, as argued by Easley and O'Hara (2004). They define the "float" of an asset as the actual number of assets available for trade in a given period.

2.2 Equilibrium bond prices

Equating aggregate demand $\int \alpha_t^j dj$ and supply $\mathbf{s}_t$

$$\Sigma_p^{-1} v_t = \Sigma_p^{-1} \left(\int E \left[\mathbf{p}_{t+1}^{-} \mid \Omega_t^j \right] dj - \mathbf{p}_t - \mathbf{r}_t \right) + \frac{1}{2} \Sigma_p^{-1} diag \left[\Sigma_p \right] \tag{2.7}$$

and solving for the vector of current log bond prices $\mathbf{p}_t$ gives

$$\mathbf{p}_t = \frac{1}{2} diag \left[\Sigma_p \right] - \mathbf{r}_t + \int E \left[\mathbf{p}_{t+1}^{-} \mid \Omega_t^j \right] dj - v_t \tag{2.8}$$

A generic element of $\mathbf{p}_t$ is thus the log price p_t^n of an n periods to maturity zero coupon bond given by

$$p_t^n = \frac{1}{2} \sigma_n^2 - r_t + \int E \left[p_{t+1}^{n-1} \mid \Omega_t^j \right] dj - v_t^n \tag{2.9}$$

where $\frac{1}{2}\sigma_n^2$ and v_t^n are the relevant elements of $\frac{1}{2} diag \left[\Sigma_p \right]$ and v_t respectively. The price of an n periods to maturity bond in period t thus depends on the average expectation in period t of the price of a $n-1$ period bond in period $t+1$. The more an agent expects to be able to sell a bond for in the future, the more is he willing to pay for it today. However, risk aversion prevents the most optimistic agent from demanding all of the available supply.

2.3 The term structure of interest rates as a function of higher order expectations

As usual, we can start from the one period risk-free bond

$$p_t^1 = -r_t \tag{2.10}$$

and apply (2.9) recursively. The log price of a two period bond is then given by

$$p_t^2 = \frac{1}{2} \sigma_2^2 - r_t - \int E \left[r_{t+1} \mid \Omega_t^j \right] dj + v_t^2 \tag{2.11}$$

i.e. p_t^2 is a function of the average first order expectations about the next period risk free rate r_t but not of higher order expectations.

The price of a three period bond according to (2.9) is given by the average expectation of the price of a two period bond in $t+1$, discounted by the short rate r_t. Leading (2.11) by one period and substituting into (2.9) with $n = 3$ gives

$$\begin{aligned} p_t^3 \quad = \quad & \frac{1}{2} \left(\sigma_2^2 + \sigma_3^2 \right) - r_t - \int E \left[r_{t+1} \mid \Omega_t^j \right] dj \\ & - \int E \left[\int E \left[r_{t+2} \mid \Omega_{t+1}^{j'} \right] dj' \mid \Omega_t^j \right] dj \\ & + v_t^3 \end{aligned} \tag{2.12}$$

so that the three period bond is a function of average first and second order expectations about future risk free rates.

Applying the same procedure recursively to derive the price of an n periods to maturity bond gives

$$\begin{aligned} p_t^n &= \frac{1}{2}\sum_{i=2}^{n}\sigma_i^2 - r_t - \int E\left[r_{t+1} \mid \Omega_t^j\right] \\ & -\int E\left[\int E\left[r_{t+2} \mid \Omega_{t+1}^{j'}\right] dj' \mid \Omega_t^j\right] dj - ... \\ & ... - \int E\left[\int E\left[... \int E\left[r_{t+n-1} \mid \Omega_{t+n-2}^{j''}\right] dj'' ... \mid \Omega_{t+1}^{j'}\right] dj' \mid \Omega_t^j\right] dj + v_t^n \end{aligned} \tag{2.13}$$

This expressions corresponds to Eq (15) in the main text, but with restriction that $m_{t+1} = -r_t - v_t$.

2.4 Bond price recursions in the equilibrium model

We can now show that the equilibrium model results in bond prices of the form (1.6) and (1.7).

2.4.1 Full information

Under full information, the price of an n period bond in the equilibrium model

$$p_t^n = \frac{1}{2}\sigma_n^2 - r_t + \int E\left[p_{t+1}^{n-1} \mid \Omega_t^j\right] dj - v_t^n \tag{2.14}$$

simplifies to

$$p_t^n = \frac{1}{2}\sigma_n^2 - r_t + E\left[p_{t+1}^{n-1} \mid x_t\right] - v_t^n \tag{2.15}$$

where the covariance term σ_n^2 is given by

$$\sigma_2^2 = \delta_x CC'\delta_x \tag{2.16}$$

for $n = 2$ (there is no maturity specific shock in p_t^1) and more generally for $n > 2$

$$\sigma_n^2 = B_{n-1}'CC'B_{n-1} + V_{n-1}V_{n-1}' \tag{2.17}$$

Starting the recursion for p_t^n from

$$p_t^1 = -r_t \tag{2.18}$$

so that

$$A_1 = -\delta_0 \tag{2.19}$$

and

$$B_1 = -\delta_x \tag{2.20}$$

we get

$$p_t^2 = \frac{1}{2}\sigma_n^2 - r_t + A_1 + B_1'E\left[x_{t+1} \mid x_t\right] - v_t^2 \tag{2.21}$$

which can be rearranged to

$$p_t^2 = \frac{1}{2}\sigma_2^2 - \delta_0 - \delta_x + A_1 + B_1'\mu^P + B_1'F^P x_t - v_t^2 \tag{2.22}$$

so that

$$\begin{aligned} A_2 &= 2\delta_0 + \frac{1}{2}\delta_x CC'\delta_x + B_1'\mu^P & (2.23)\\ B_2' &= \delta_x x_t + B_1'F^P x_t & (2.24) \end{aligned}$$

Continued substitution yields the general expressions

$$\begin{aligned} p_t^n &= A_{n-1} - \delta_0 + B_{n-1}'\mu^P + \frac{1}{2}\left(B_{n-1}'CC'B_{n-1} + V_{n-1}V_{n-1}'\right) & (2.25)\\ &\quad -\delta_x x_t + B_{n-1}'F^P x_t & (2.26) \end{aligned}$$

Substituting in the conjectured form (1.6) and equating coefficients gives

$$\begin{aligned} A_n &= A_{n-1} - \delta_0 + B_{n-1}'\mu^P + \frac{1}{2}\left(B_{n-1}'CC'B_{n-1} + V_{n-1}V_{n-1}'\right) & (2.27)\\ B_n' &= -\delta_x + B_{n-1}'F^P & (2.28) \end{aligned}$$

2.4.2 Heterogeneous information

Under heterogenous information, the price of an n period bond in the equilibrium model

$$p_t^n = \frac{1}{2}\sigma_n^2 - r_t + \int E\left[p_{t+1}^{n-1} \mid \Omega_t^j\right] dj - v_t^n \tag{2.29}$$

can be written as

$$p_t^n = \frac{1}{2}\sigma_n^2 - r_t + A_{n-1} + B_{n-1}^{'}\int E\left[X_{t+1} \mid \Omega_t^j\right] dj - v_t^n \tag{2.30}$$

As usual, we can start the recursions from

$$\begin{aligned} p_t^1 &= -r_t & (2.31)\\ &= -\delta_0 - \delta_X X_t & (2.32) \end{aligned}$$

so that

$$\begin{aligned} A_{n+1} &= -\delta_0 + A_n + B_n'\mu^X + \frac{1}{2}\left[B_n'\Sigma_{t+1|t}B_n' + V_nV_n'\right] & (2.33)\\ &\quad B_n'\mathcal{C}V_n' \end{aligned}$$

and

$$B_{n+1}' = -\delta_X' + B_n'\mathcal{F}H \tag{2.34}$$

where X_{t+1} is defined as in the main text. To derive the expressions (2.33) and (2.34) we used that

$$\int E\left[X_{t+1} \mid \Omega_t^j\right] dj = \mathcal{F}HX_t \tag{2.35}$$

and that

$$\sigma_n^2 = B_n' \Sigma_{t+1|t} B_n' + V_n V_n' + 2B_n' \mathcal{C} V_n' \tag{2.36}$$

where $\Sigma_{t+1|t}$ is the covariance of the one period ahead conditional forecast errors about the state X_t

$$\Sigma_{t+1|t} \equiv E\left(X_{t+1} - E\left(X_{t+1} \mid \Omega_t^j\right)\right)\left(X_{t+1} - E\left(X_{t+1} \mid \Omega_t^j\right)\right)' \tag{2.37}$$

and H is the average expectations operator defined in Eq (44) in the main text.

3 A full information affine model with maturity specific shocks

Under full information, agents are assumed to observe x_t directly. Taking the conjectured bond price equation (1.6) and the law of motion (1.2) as given, the expected price of an $n-1$ period bond in period $t+1$ is then given by

$$E\left(p_{t+1}^{n-1} \mid x_t\right) = A_{n-1} + B_{n-1}'\left(\mu^{\mathbb{P}} + F^{\mathbb{P}} x_t\right) \tag{3.1}$$

The forecast error is thus given by

$$p_{t+1}^{n-1} - E\left(p_{t+1}^{n-1} \mid x_t\right) \equiv B_{n-1}' C u_{t+1} + v_t^{n-1} \tag{3.2}$$

which is given by

$$\begin{aligned} p_{t+1}^{n-1} - E\left(p_{t+1}^{n-1} \mid x_t\right) &= \begin{bmatrix} B_{n-1}'C & V_{n-1} \end{bmatrix} u_{t+1} && (3.3) \\ &\equiv \psi_{n-1} u_{t+1} && (3.4) \end{aligned}$$

In the full information model, the vector u_{t+1} thus spans the risk of holding bonds that agents want to be compensated for. We specify the natural logarithm of the SDF to take the form

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t' u_{t+1} : u_{t+1} \sim N(0, I) \tag{3.5}$$

and the market price of risk λ_t as

$$\lambda_t = \lambda_0 + \lambda_x x_t + \lambda_v v_t \tag{3.6}$$

Apart from the dependence on the maturity specific shocks v_t, this specification is identical to the standard full information affine model. We will now show how the SDF (3.5) together with the no-arbitrage condition

$$p_t^n = \log E\left[\exp\left(m_{t+1} + p_{t+1}^{n-1}\right) \mid x_t\right] \tag{3.7}$$

result in bond pricing equations of the conjectured form (1.6).

3.1 Bond price recursions

Start by substituting in the stochastic discount factor (3.5) into the no arbitrage condition

$$p_t^n = \log E\left[\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t u_{t+1} + p_{t+1}^{n-1}\right) \mid x_t\right] \tag{3.8}$$

Then use the conjectured bond price equation (1.6) to replace p_{t+1}^{n-1}

$$p_t^n = \log E\left[\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t' u_{t+1} + E\left(p_{t+1}^{n-1} \mid x_t\right) + \psi_{n-1}u_{t+1}\right) \mid x_t\right] \tag{3.9}$$

Taking all terms known at time t outside the expectations operator

$$\begin{aligned} p_t^n &= -r_t - \frac{1}{2}\lambda_t'\lambda_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t \\ &\quad + \log E\left[\exp\left(\left[\psi_{n-1} - \lambda_t'\right] u_{t+1}\right) \mid x_t\right] \end{aligned} \tag{3.10}$$

and use that the term in brackets is a log-normally distributed random variable to get

$$\begin{aligned} p_t^n &= -r_t - \frac{1}{2}\lambda_t'\lambda_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t \\ &\quad + \frac{1}{2}\left(\psi_{n-1} - \lambda_t'\right)\left(\psi_{n-1} - \lambda_t'\right)' \end{aligned} \tag{3.11}$$

$$\begin{aligned} p_t^n &= -r_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t \\ &\quad + \frac{1}{2}\psi_{n-1}\psi_{n-1}' - \psi_{n-1}\lambda_t \end{aligned} \tag{3.12}$$

Finally, substituting in λ_t gives

$$\begin{aligned} p_t^n &= -r_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t \\ &\quad + \frac{1}{2}\psi_{n-1}\psi_{n-1}' - \psi_{n-1}\left(\lambda_0 + \lambda_x x_t + \lambda_v v_t\right) \end{aligned} \tag{3.13}$$

and expanding

$$\begin{aligned} p_t^n &= -r_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t + \frac{1}{2}\psi_{n-1}\psi_{n-1}' - \psi_{n-1}\lambda_0 \\ &\quad -\psi_{n-1}\lambda_x x_t - \psi_{n-1}\lambda_v v_t \end{aligned} \tag{3.14}$$

Using the conjectured bond price function (1.6) to replace p_t^n

$$\begin{aligned} A_n + B_n' x_t + v_t^n &= -\delta_0 - \delta_x x_t + A_{n-1} + B_{n-1}'\mu^P + B_{n-1}'F^P x_t + \frac{1}{2}\psi_{n-1}\psi_{n-1}' - \psi_{n-1}\lambda_0 \\ &\quad -\psi_{n-1}\lambda_x x_t - \psi_{n-1}\lambda_v v_t \end{aligned} \tag{3.15}$$

and matching terms

$$A_n = -\delta_0 + A_{n-1} + B_{n-1}'\mu^P + \frac{1}{2}\psi_{n-1}\psi_{n-1}' - \psi_{n-1}\lambda_0 \tag{3.16}$$

$$B_n' = -\delta_x + B_{n-1}'F^P - \psi_{n-1}\lambda_x \tag{3.17}$$

$$v_t^n = -\psi_{n-1}\lambda_v v_t \tag{3.18}$$

3.2 The maturity specific shocks

To get the conjectured bond price equation of the form (1.6) we also need find an λ_v such that the equality

$$v_t^n = -\psi_{n-1}\lambda_v v_t \tag{3.19}$$

holds for each n. Start by stacking the expression fo each n on top of each other so that

$$v_t = -\Psi\lambda_v v_t \tag{3.20}$$

$$\Psi = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_{\overline{n}-1} \end{bmatrix} \tag{3.21}$$

Setting

$$\lambda_v = -\Psi_{right}^{-1} \tag{3.22}$$

where $(\cdot)_{right}^{-1}$ denotes the (right) one-sided inverse of a matrix then ensures that equation (3.19) holds for each n. The (right) one-sided inverse of Ψ exists as long as Ψ is of full rank, i.e. equal to $\overline{n}$. This condition is always fulfilled in the full information model since

$$\Psi = \begin{bmatrix} B_1' & 0 \\ B_2' & V_2 \\ \vdots & \vdots \\ B_{\overline{n}-1}' & V_{\overline{n}-1} \end{bmatrix} \tag{3.23}$$

is of rank $\overline{n} - 2$ and $B_1 \neq 0$. Letting λ_v be defined this way also implies that the presence of maturity specific shocks does not introduce any additional parameters through λ_v.

4 The equilibrium model as a restricted special case of the affine model

4.1 Full information

Comparing the expressions (3.16) - (3.18) in the affine model with the corresponding expressions (2.27) - (2.28) of the equilibrium model shows that imposing the restrictions

$$\lambda_0 = \mathbf{0} \tag{4.1}$$

$$\lambda_x = \mathbf{0} \tag{4.2}$$

$$\lambda_v = -\Psi_{right}^{-1} \tag{4.3}$$

on the affine model gives the same bond price equations as the equilibrium model. The equilibrium model described in the Appendix is thus a nested (and restricted) special case of the affine no-arbitrage model under full information.

4.2 Heterogeneous information

It is now straightforward to verify that the equilibrium model with heterogeneous information is a nested special case of the affine heterogeneous information model in the main paper. The corresponding restrictions on the expressions from the affine heterogeneous information model in the paper are given by

$$\Lambda_0 = \mathbf{0} \tag{4.4}$$

$$\widehat{\Lambda}_x = \mathbf{0} \tag{4.5}$$

$$\Lambda_n = -\Sigma_a^{-1} \tag{4.6}$$

Substituting (4.4) - (4.5) into the recursions

$$A_{n+1} = -\delta_0 + A_n + B_n' \mu_X + \frac{1}{2} e_n' \Sigma_a e_n - e_n' \Sigma_a \Lambda_0 \tag{4.7}$$

and

$$B_{n+1}' = -\delta_X + B_n' \mathcal{F} H - e_n' \Sigma_a \widehat{\Lambda}_x \tag{4.8}$$

from the main paper and simplifying gives

$$\begin{aligned} A_{n+1} &= -\delta_0 + A_n + B_n' \mu^X + \frac{1}{2} \left[B_n' \Sigma_{t+1|t} B_n + V_n V_n' \right] \\ &\quad -\Sigma_n \Lambda_0 + B_n' \mathcal{C} V_n' \end{aligned} \tag{4.9}$$

$$B_{n+1}' = -\delta_X' + B_n' \mathcal{F} H \tag{4.10}$$

which corresponds to the recursions (4.7) - (4.8) in the equilibrium model above since

$$e_n' \Sigma_a e_n = B_n' \Sigma_{t+1|t} B_n + V_n V_n' + 2 B_n' \mathcal{C} V_n' \tag{4.11}$$

The affine model with heterogeneous information and maturity specific shocks thus nests, and is therefore consistent, with an equilibrium model with heterogeneously informed agents that make explicit and trading and portfolio decisions.

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