

# OPTIMAL MONETARY POLICY WITH REAL TIME SIGNAL EXTRACTION FROM THE BOND MARKET

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**ABSTRACT.** Monetary policy is conducted in an environment of uncertainty. This paper sets up a model where the central bank uses real time data from the bond market together with standard macroeconomic indicators to estimate the current state of the economy more efficiently, while taking into account that its own actions influence the bond market and therefore what it observes. The timeliness of bond market data allows for quicker monetary policy responses to disturbances and reduces the variance of inflation and the output gap as compared to the case when the central bank has to rely solely on collected aggregate data. That the central bank uses the information in the term structure to set policy creates a link between the bond market and the macro economy that is novel to the literature. To quantify the importance of the bond market as a source of information, the model is estimated on data for the United States using Bayesian methods. The empirical exercise suggests that there is some information in US yields of maturities of less than one year that can help the Federal Reserve to identify shocks to the economy on a timely basis. **Keywords:** Monetary Policy, Imperfect Information, Bond Market, Term Structure of Interest Rates

## 1. INTRODUCTION

Hayek (1945) famously argued that market economies are more efficient than planned economies because of markets' ability to efficiently use information dispersed among market participants. In most western economies there is now little planning and almost all prices are determined by market forces without interference from any central authority. However, there is one important exception: the market for short term nominal debt where central banks borrow and lend at fixed interest rates. In the presence of nominal frictions in product or wage markets, this practise can improve welfare by reducing the volatility of inflation and output. Hayek's insight, though formulated in a more general setting of a planned economy, was that even a central bank that shares the objective of the representative agent may not be *able* to implement an optimal stabilizing policy due to incomplete information. In this paper, the central bank would implement an optimal stabilizing policy if it knew the state of the economy with certainty, and any deviation from optimal policy is due only to information imperfections. Under this assumption we demonstrate how the central bank can make use of

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Hayek’s insight and use the market for debt of longer maturities as a source of information that makes a more efficient estimation of the state of the business cycle possible, and thus reduces deviations from optimal policy. That this is close to how some central banks think about and use the term structure is illustrated by a quote by the Chairman (then Governor) of the Federal Reserve Board, Ben Bernanke:

*“To the extent that financial markets serve to aggregate private-sector information about the likely future course of inflation, data on asset prices and yields might be used to validate and perhaps improve the Fed’s forecasts.”*<sup>1</sup>

The suggestion that the bond market can provide information that is valuable to policy makers is thus not news to the policy makers themselves. Rather, the contribution of the present paper is to provide a coherent framework for analyzing and estimating the interaction between information contained in the term structure and the monetary policy making process. In the model presented below the central bank set interest rates in an uncertain environment, where the yield curve is informative about the state of the economy and thus also informative about the desired interest rate. This has the consequence that the macro economy is not independent of the term structure. The only direct effect of interest rates on the macro economy is from the expected path of the short rate set by the central bank to aggregate demand, as is standard in the New-Keynesian literature. However, there is also an indirect feedback from rates on longer maturity bonds to the macro economy through an information channel. The mechanism is the following. Bonds are traded daily and the affine form of the bond pricing function makes the bond pricing equation with macro factors formally equivalent to a linear measurement of the state of economy. The term structure can thus be used as a more timely indicator of the state of the economy than collected aggregate information that is available only with delay and sometimes significant measurement error. A movement in the term structure signals a shift in the underlying macro factors that induces the central bank to re-evaluate what the optimal short term interest rate should be. The shift in the term structure thus feeds into a change in demand through the change in the short term interest rate.

In the present model the policy makers exploit the fact that bond market participants’ expectations about the future are revealed by the term structure. As pointed out by Bernanke and Woodford (1997), letting monetary policy react mechanically to expectations may lead to a situation where expectations become uninformative about the underlying state and no equilibrium exists. They further argue that “targeting expectations” by policymakers can not be a substitute to structural modelling. In the proposed framework below, the information in the term structure is *complementary* to other information and firmly connected to an underlying structural model. Policymakers then avoid the potential pitfalls of a pure “expectations targeting” regime.

There is a large literature on the informational content of the term structure. Mostly, it has focused on whether the term structure, often modelled as the spread between short and long rates, can help predict future outcomes of macro variables.<sup>2</sup> This literature thus

<sup>1</sup>“What Policy Makers Can Learn from Asset Prices”, Remarks by Governor Ben S. Bernanke before *The Investment Analysts Society of Chicago*, Chicago, Illinois, April 15th, 2004.

<sup>2</sup>For example Harvey (1990), Mishkin (1990) and Estrella and Mishkin (1998).

investigates whether the component of the yield curve that is orthogonal to other macro variables adds predictive power to forecasting models of inflation and GDP growth. A recent paper in this vein is Ang, Piazzesi and Wei (2003) who finds that the short interest rate performs better than any term spread in predicting GDP growth. The negative correlation between the orthogonal component of short interest rates and future output found by Ang et al is consistent with the findings of the VAR literature on the real effects of monetary policy shocks. In the VAR literature on the transmission mechanism of monetary policy, the monetary policy shocks are defined to be exactly the component of interest rates that is orthogonal to other macro variables (e.g. Christiano, Eichenbaum and Evans (2001)). It is thus not surprising that the previous literature found that including the slope of the yield curve, which will be negatively correlated with the orthogonal monetary policy shock, improves forecasts. In this paper we deviate from this literature on the informational content of the term structure by asking if there is information in the term structure that can be used in the monetary policy process when *the transmission mechanism is assumed to be known and the effect of the short rate is controlled for*.

There are two potentially important types of information that could be revealed by the term structure that the present model is silent about. Goodfriend (1998) discusses the Federal Reserve's responses to "inflation scares" in the 1980s, which he defines as increases in the long term yields. He interprets these as doubts by market participants about the Federal Reserve's commitment to fighting inflation. The present paper does not address questions about central bank credibility, but takes a perfectly credible central bank with a publicly known inflation target as given. The model presented here is also not suited to analyzing or interpreting market perceptions of the reasons for a change in the monetary policy stance, as done by Ellingsen and Söderström (2001). The policymakers' relative preferences for stabilizing inflation or the output gap are assumed to be publicly known. In this paper, we restrict our attention to what the term structure can tell us about the state of the business cycle.

The practical relevance of any information contained in the term structure is ultimately an empirical question. When bond markets are noisy, observing the term structure is not very informative. In order to quantify the informational content of the term structure the variances of the non-fundamental shocks in the term structure are estimated simultaneously with the structural parameters of the macro economy. The estimation methodology is similar to recent work by Hördahl, Tristani and Vestin (2004) who estimate the term structure dynamics jointly with a small empirical macro model where the central bank is assumed to be perfectly informed. Hördahl *et al* impose only a no arbitrage condition on the pricing of bonds while in this paper the bond pricing function and the dynamics of the macro economy are derived from the same underlying utility function. This makes the analysis more stringent, but it comes at the cost of an empirically less flexible bond pricing function.

In the next section a model is presented where the central bank extracts information from the term structure about unobservable shocks while recognizing that its own actions influence the term structure itself. In Section 3 the model is estimated to quantify the potential of the yield curve as a source of information. Section 4 concludes.

## 2. THE MACRO ECONOMY, THE TERM STRUCTURE AND MONETARY POLICY UNDER IMPERFECT INFORMATION

This section presents a model where the central bank extracts information about the state of the economy from the term structure of interest rates. Movements in the term structure will then have a direct impact on the macro economy through its effect on the central bank's estimate of the state and therefore also on the setting of the policy instrument. This means that the macro economy, the term structure model and the filtering problem of the central bank have to be solved simultaneously which makes the model different from other recent papers, for example Hördahl *et al* (2004) and Bekaert, Cho and Moreno (2005), where the macro economic model can be solved separately from the term structure. Though the complete model has to be solved simultaneously, it is still useful to divide the description of the model into three parts. The macro economy is described first without specifying an explicit interest rate function, but merely noting that it is set by the central bank to minimize a loss function that in principle could be derived from micro foundations. The filtering problem of the central bank is then solved, taking the term structure model as given. Finally, in the last part the term structure model is derived.

**2.1. The Macro Economy.** We use a standard business cycle model of the macro economy with monopolistically competitive firms that sell differentiated goods. Prices are set according to the Calvo mechanism, with a fraction of firms using a rule of thumb rather than optimizing as in Galí and Gertler (1999). Households supply labor and consume goods. In addition to their own current consumption, they also care about the lagged aggregate consumption level.

**2.1.1. Households and firms.** Consider a representative household  $j \in (0, 1)$  that wishes to maximize the discounted sum of expected utility

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s U(C_{t+s}(j), N_{t+s}(j)) \right\} \quad (1)$$

where  $\beta \in (0, 1)$  is the household's subjective discount factor and the period utility function in consumption  $C_t$  and labor  $N_t$  is given by

$$U(C_t(j), N_t(j)) = \frac{(C_t(j)H_t^{-\eta})^{1-\gamma}}{(1-\gamma)} - \frac{N_t(j)^{1+\varphi}}{1+\varphi}. \quad (2)$$

The variable  $H_t$

$$H_t = \int C_{t-1}(j) dj \quad (3)$$

is a reference level of consumption that we interpret as external habits that makes marginal utility of consumption an increasing function of lagged aggregate consumption. The habit specification helps to explain the inertial movement of aggregate output as well as the procyclicality of asset prices.<sup>3</sup> Differentiated goods indexed by  $i \in (0, 1)$  are produced with a

<sup>3</sup>See Campbell and Cochrane (1999) for the implications of habits for asset prices.

technology that is linear in labor and subject to a persistent productivity shock  $A_t$

$$Y_t(i) = A_t N(i) \quad (4)$$

that follow an AR(1) process in logs

$$a_t = \rho a_{t-1} + \varepsilon_t^a \quad (5)$$

$$\varepsilon_t^a \sim N(0, \sigma_{a\varepsilon}^2). \quad (6)$$

Firms set prices according to the Calvo (1983) mechanism where a fraction  $\theta$  of firms reset their price in a given period. Of the firms resetting their price, a fraction  $(1 - \omega)$  optimize their price decision and take into account that their price may be effective for more than one period while a fraction  $\omega$  of price setters use a “rule of thumb” as in Galí and Gertler (1999). Price setters using a “rule of thumb” set their price equal to last period’s average reset price plus the lagged inflation rate.

2.1.2. *The linearized model.* The linearized structural equations are given by equations (7) - (8)

$$y_t = \frac{\gamma}{\gamma - \eta + \gamma\eta} E_t y_{t+1} + \frac{-\eta(1 - \gamma)}{\gamma - \eta + \gamma\eta} y_{t-j} - \frac{1}{\gamma - \eta + \gamma\eta} [i_t - E_t \pi_{t+1}] + \varepsilon_t^y \quad (7)$$

$$\begin{aligned} \pi_t = & \frac{\beta\theta}{\theta + \omega(1 - \theta(1 - \beta))} E_t \pi_{t+1} + \frac{\omega}{\theta + \omega(1 - \theta(1 - \beta))} \pi_{t-1} \\ & + \frac{(1 - \omega)(1 - \theta)(1 - \theta\beta)}{\theta + \omega(1 - \theta(1 - \beta))} mc_t + \varepsilon_t^\pi \end{aligned} \quad (8)$$

where  $\{y_t, \pi_t, i_t\}$  is real output, inflation and the short nominal interest rate in period  $t$ . Marginal cost in period  $t$ ,  $mc_t$ , can be found by equating the marginal utility of consuming the real wage paid for an additional unit of labor with the household’s disutility of providing the additional unit of labor. The real marginal cost then equals the market clearing real wage divided by productivity

$$mc_t = (\varphi + \gamma) y_t + \eta(1 - \gamma) y_{t-1} - (1 + \varphi) a_t. \quad (9)$$

where the relationship

$$n_t = y_t - a_t \quad (10)$$

was used to substitute out labor supply. Potential output,  $\bar{y}_t$ , defined as the level of output that is compatible with no acceleration in inflation then is

$$\bar{y}_t \equiv \frac{\eta(1 - \gamma)}{(\varphi + \gamma)} y_{t-1} + \frac{1 + \varphi}{(\varphi + \gamma)} a_t. \quad (11)$$

The short term interest rate is set by a monetary authority to minimize the expected value of the loss function

$$\mathbf{L}_t = E_t \left[ \sum_{k=0}^{\infty} \beta^k [\lambda_y (y_{t+k} - \bar{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_{t+k} - i_{t+k-1})^2] \right]. \quad (12)$$

The weights  $\lambda_y$  and  $\lambda_i$  can be chosen such that the loss function (12) is a second order approximation of the utility function of the representative agent.<sup>4</sup> However, we do not necessarily want to impose this restriction when we estimate the model. The equations (5), (7), (8) and (9) can be written more compactly as

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = A \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + B i_t + C \varepsilon_t \quad (13)$$

where

$$X_{1,t} = [a_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, i_{t-1}, \Delta i_t, \mathbf{v}_t]' \quad (14)$$

$$X_{2,t} = [y_t, \pi_t]' \quad (15)$$

$$\varepsilon_t = [\varepsilon_t^a, \varepsilon_t^y, \varepsilon_t^\pi, \mathbf{e}_t^v]', \quad \varepsilon_t \sim N(0, \Sigma) \quad (16)$$

That the innovations vector  $\varepsilon_t$  is assumed to be normally distributed will be used later. The vector  $\mathbf{v}_t$  in  $X_{1,t}$  are (potentially serially correlated) measurement errors that follow

$$\mathbf{v}_t = W \mathbf{v}_{t-1} + \mathbf{e}_t^v \quad (17)$$

The measurement errors have not yet been introduced to the model but are included in the state definition already here in order to keep notation compact and consistent throughout the paper.

**2.2. Monetary Policy and Real Time Signal Extraction.** Monetary policy operates in an uncertain environment where some state variables are only observed with error and delay and some variables, like productivity and thus potential output, are not observed at all. Variables that are not observable but relevant for monetary policy have to be inferred from the variables that are observable. In such a setting, Svensson and Woodford (2003) show that a form of certainty equivalence holds. That is, with a quadratic objective function and linear constraints, the optimal interest rate can be expressed as a linear function of the central bank's estimate of the state  $X_{1,t|t}$

$$X_{1,t|t} = E_t [X_{1,t} | I_t]$$

where  $I_t$  is the information set of the central bank at time  $t$ . The coefficients in the policy function are then the same as they would be if the central bank could observe the predetermined state perfectly. The coefficient vector  $F$  of the optimal interest function

$$i_t = F X_{1,t|t} \quad (18)$$

can thus be found by standard full information methods, for instance by the algorithm in Söderlind (1999). Here we describe how the central bank can apply the Kalman filter to estimate the state  $X_{1,t}$ . The affine function that maps the predetermined state into bond prices, characterized by the matrices  $Q_1$  and  $Q_2$ , are taken as given and deriving the equilibrium dynamics of the model is then a straightforward application of the procedure in Svensson and Woodford (2003).

<sup>4</sup>See Amato and Laubach (2004).

Partition the coefficient matrices in (13) conformably to the predetermined and forward looking variables and substitute in the interest rate function to get

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} F X_{1,t|t} + C \varepsilon_t. \quad (19)$$

The equilibrium dynamics of the model can then be described by equations (20)-(24)

$$X_{1,t} = H X_{1,t-1} + J X_{1,t-1|t-1} + C_1 \varepsilon_t \quad (20)$$

$$X_{2,t} = G^1 X_{1,t} + (G - G^1) X_{1,t|t} \quad (21)$$

$$X_{1,t|t} = X_{1,t|t-1} + K [Z_t - L_1 X_{1,t|t-1} - L_2 X_{1,t|t}] \quad (22)$$

$$Z_t = \mathbf{z} + L_1 X_{1,t} + L_2 X_{1,t|t} \quad (23)$$

$$\mathcal{Y}_t = \mathbf{q} + Q_1 X_{1,t} + Q_2 X_{1,t|t} \quad (24)$$

where  $Z_t$  is the vector of variables that are observable to the central bank and  $\mathcal{Y}_t$  is a vector of bond yields of different maturities. The system of equations (20)-(24) can be written solely as functions of the actual state, the central bank's estimate of the state and the shock vectors  $\varepsilon_t$ . The coefficient matrices  $G$  and  $G^1$  are derived in Svensson and Woodford (2003) and satisfy equations (25) and (26)

$$G = (A_{22} - G A_{12})^{-1} [-A_{21} + G A_{11} + (G B_1 - B_2) F] \quad (25)$$

$$G^1 = A_{22}^{-1} \{-A_{21} + [G^1 + (G - G^1) K L_1] H\} \quad (26)$$

where the following definitions were used

$$H \equiv A_{11} + A_{12} G^1 \quad (27)$$

$$J \equiv B_1 F + A_{12} (G - G^1). \quad (28)$$

The Kalman gain matrix  $K$  is given by

$$K = P L_1' (L_1 P L_1')^{-1} \quad (29)$$

$$P = H (P - P L_1' (L_1 P L_1')^{-1} L_1 P) H' + \Sigma \quad (30)$$

where  $P$  is the one period ahead forecast error. The coefficient matrices  $G, G^1$ , the Kalman gain  $K$  and the one period ahead state forecast error  $P$  have to be determined jointly by finding a fixed point of the system described by the equations (25),(26),(29) and (30). Before we can solve for the equilibrium dynamics we need to specify the selection matrices  $L_1$  and  $L_2$  in the observation equation (23). We thus have to decide what the central bank can observe.

*2.2.1. Variables observable by the central bank.* The central bank observes bond yields contemporaneously, while output and inflation are only observable with a one period lag. This is a compromise that is necessary due to the division of time into discrete periods that do not conform to the exact delays of data releases, though it does capture some broad features of data availability. Data on real GDP are released with a significant delay while bond prices are observed every day that bonds are traded. The compromise is the observation of the price level. In most countries, CPI data are released the month after observation so the one

quarter lag is thus too long for most countries. We can write the measurement equation (23) as

$$Z_t = \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \mathcal{Y}_t \end{bmatrix} + \begin{bmatrix} v_t^y \\ v_t^\pi \\ \mathbf{0} \end{bmatrix} \quad (31)$$

and the matrices  $L_1$  and  $L_2$  are then given by

$$L_1 = \begin{bmatrix} \mathbf{0}_{2 \times 1} & I_2 & \mathbf{0}_{2 \times 4} & I_2 & \mathbf{0}_{2 \times 3} \\ & & Q_1 & & \end{bmatrix} \quad (32)$$

$$L_2 = \begin{bmatrix} \mathbf{0}_{2 \times 12} \\ Q_2 \end{bmatrix}. \quad (33)$$

The information set of the central bank is given by

$$I_t^{cb} \equiv \{A, B, C, Q_1, Q_2, \Sigma_{\varepsilon\varepsilon}, Z_{t-s} \mid s \geq 0\} \quad (34)$$

that is, in addition to observing the vector  $Z_t$ , the central bank also knows the structure of the economy.

**2.3. The Law of Motion for the State of the Economy.** In full information models, the relevant state for the pricing of bonds is the same as the state of the economy. In the present model the central bank cannot observe the state of the economy with certainty and uses the Kalman filter to estimate it. The central bank's information set is a subset of the information set of the bond market participants, who are assumed to observe the state perfectly. This assumption allows us to model bond market participants as if they know the central bank's estimate of the state. We define the extended state  $\bar{X}_t$  as

$$\bar{X}_t \equiv [ X_{1,t} \quad X_{1,t|t} ]' \quad (35)$$

and we want to find a system of the form

$$\bar{X}_t = M\bar{X}_{t-1} + N\varepsilon_t \quad (36)$$

$$\mathcal{Y}_t = \mathbf{q} + Q\bar{X}_t \quad (37)$$

that is, we conjecture that yields  $\mathcal{Y}_t$  are an affine function of the extended state  $\bar{X}_t$ . We start by substituting the observation equation (23) into the central banks updating equation (22) to get

$$X_{1,t|t} = X_{1,t|t-1} + K [L_1 X_{1,t} + L_2 X_{1,t|t} - L_1 X_{1,t|t-1} - L_2 X_{1,t|t}] \quad (38)$$

Using equations (20)-(22) and the definitions (27) and (28) and rearranging we get

$$\begin{aligned} X_{1,t|t} = & [(H + J) + KL_1 J - KL_1(H + J)] X_{1,t-1|t-1} \\ & + KL_1 H X_{1,t-1} + KL_1 \varepsilon_t \end{aligned} \quad (39)$$

The matrices  $K$ ,  $L_1$  and  $L_2$  depend on the coefficients in the conjectured term structure function (37) and the covariance matrix of the structural shocks and measurement errors

$\Sigma_{\varepsilon\varepsilon}$ . Combining equation (20) and (39) we get the conjectured form from equation (36)

$$\begin{aligned} \bar{X}_t = & \begin{bmatrix} H & J \\ KL_1H & [(H+J) + KL_1J - KL_1(H+J)] \end{bmatrix} \bar{X}_{t-1} \\ & + \begin{bmatrix} C_1 \\ KL_1C_1 \end{bmatrix} \varepsilon_t \end{aligned} \quad (40)$$

**2.4. The Term Structure and the State of the Economy.** In this section we derive the law of motion for the nominal stochastic discount factor that is used to price bonds. We use a similar strategy as Bekaert, Cho and Moreno (2005), and like them, we make use of the fact that the normality of the shocks in the linearised model makes the stochastic discount factor implied by the structural model log normal. This allows the term structure to be derived as an affine function of the state of the economy.

Define the nominal stochastic discount factor  $E_tM_{t+1}$  as

$$E_tM_{t+1} \equiv E_t\beta \frac{U_{ct+1}P_t}{U_{ct}P_{t+1}}. \quad (41)$$

where  $U_{ct}$  is the marginal utility of consumption in period  $t$ . That the price in period  $t$  of a dollar delivered in period  $t+n$  is given by  $E_tM_{t+1}M_{t+2}\dots M_{t+n}$  can be used to derive a recursive formula for the term structure. Plug in the utility function (2) into (41) and take logs to get

$$\log E_tM_{t+1} = \log \beta - \gamma E_t c_{t+1} + (\gamma - \eta + \gamma\eta)c_t + \eta(1 - \gamma)c_{t-1} - E_t\pi_{t+1} - \frac{\sigma_m^2}{2} \quad (42)$$

where we used that a log of the expectation of a log normal variable equals the expectation of the log minus half its conditional variance, i.e.

$$\log E_tM_{t+1} = E_tm_{t+1} - \frac{\sigma_m^2}{2} \quad (43)$$

and that

$$m_{t+1} = -\gamma c_{t+1} + (\gamma - \eta + \gamma\eta)c_t + \eta(1 - \gamma)c_{t-1} - \pi_{t+1} \quad (44)$$

In equilibrium

$$E_tm_{t+1} + i_t = 0 \quad (45)$$

must hold and we know from (18) that (the deviation of) the interest rate (from its mean)  $i_t$  is a function of the state  $\bar{X}_t$

$$i_t = \bar{F}\bar{X}_t \quad (46)$$

$$\bar{F} = \begin{bmatrix} \mathbf{0}_{1 \times 7} & F \end{bmatrix} \quad (47)$$

which imply that the log of the expected value of the discount factor  $M_{t+1}$  is

$$\log E_tM_{t+1} = \log \beta - \bar{F}\bar{X}_t - \frac{\sigma_m^2}{2} \quad (48)$$

The conditional variance  $\sigma_m^2$  is yet to be determined. The innovation to the stochastic discount factor is given by

$$m_{t+1} - E_tm_{t+1} = V'\varepsilon_t \quad (49)$$

where the vector  $V'$

$$V' = \begin{bmatrix} -\gamma & -1 \end{bmatrix} \begin{bmatrix} G^1 & G - G^1 \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ KL_1 C_1 & K \end{bmatrix} \quad (50)$$

can be derived from equations (20) - (22) and (44). The conditional variance of  $m_{t+1}$  is then

$$\sigma_m^2 = V' \Sigma V \quad (51)$$

We can now write down an expression for the log of the price at time  $t$  of a nominal bond paying one dollar in period  $t+n$  as

$$\log \mathcal{P}_t^n = \log E_t M_{t+1} + \log E_t M_{t+2} \dots + \log E_t M_{t+n} = \bar{A}_n + \bar{B}'_n \bar{X}_t \quad (52)$$

where the constant  $\bar{A}_n$  and the vector  $\bar{B}_n$  are given by the recursive relations

$$\bar{A}_n = -\log \beta + \frac{V' \Sigma V}{2} + \bar{A}_{n-1} - \bar{B}'_{n-1} V + \frac{1}{2} \bar{B}'_{n-1} N \Sigma N' \bar{B}_{n-1} \quad (53)$$

$$\bar{B}_n = -\bar{F} + M' \bar{B}_{n-1} \quad (54)$$

starting from

$$\bar{A}_1 = -\log \beta + \frac{V' \Sigma V}{2} \quad (55)$$

$$\bar{B}_1 = -\bar{F}' \quad (56)$$

To find the vector of yields of selected maturities  $\mathcal{Y}_t$  collect the appropriate constants  $\bar{A}_n$  and vectors  $\bar{B}_n$  as

$$\mathcal{Y}_t = \begin{bmatrix} -\bar{A}_1 \\ \vdots \\ -\frac{1}{n} \bar{A}_n \end{bmatrix} + \begin{bmatrix} -\bar{B}_1 \\ \vdots \\ -\frac{1}{n} \bar{B}_n \end{bmatrix} \bar{X}_t + \mathbf{v}_t^{\mathcal{Y}} \quad (57)$$

where the yield of an  $n$  periods to maturity bond is found by dividing the price by  $n$ . Equation (57) has a dual interpretation. On one hand it can be used to express bond yields as a function of the state and the vector of shocks to the term structure,  $\mathbf{v}_t^{\mathcal{Y}}$ , are then residuals, i.e. the component of the yields that cannot be explained by the dynamics of the stochastic discount factor. A small variance of  $\mathbf{v}_t^{\mathcal{Y}}$  should then be interpreted as that the term structure model provide a good fit of the observed yields. Equation (57) can also be viewed as a measurement equation where the observable bond yields  $\mathcal{Y}_t$  tell us something about the unobservable state  $\bar{X}_t$ . The vector of shocks  $\mathbf{v}_t^{\mathcal{Y}}$  are then measurement errors and when the variance of the innovations to  $\mathbf{v}_t^{\mathcal{Y}}$  are small, the signal-to-noise ratio is high and the term structure is informative about the state of the economy. In the special case of the rank of  $[-\bar{B}_1 \ \dots \ -\frac{1}{n} \bar{B}_n]'$  being equal to the dimension of the state and  $E[\mathbf{v}_t^{\mathcal{Y}} \mathbf{v}_t^{\mathcal{Y}}] = \mathbf{0}$ , the model replicates the full information dynamics, since the state can then be backed out perfectly from the term structure. In the opposite case, when the variances of  $\mathbf{v}_t^{\mathcal{Y}}$  are very large, the model will replicate the dynamics when the central bank can only observe imperfect but direct measures of the lagged aggregate variables.

In practise, the measurement errors on the longer maturity bonds turn out to be serially correlated. By including the measurement errors in the state, it is straightforward to estimate their persistence together with the rest of the model's coefficients. The precision of the

yields as a measure of the state will then depend on the variance of the innovations  $\mathbf{e}_t^y$  to the measurement error process (17), rather than the variance of the measurement errors themselves.

Partitioning the stacked vectors  $-\frac{1}{n}\bar{B}_n$  appropriately and since  $\mathbf{v}_t^y \in \bar{X}_t$  we can write (57) in the desired form (37)

$$\mathcal{Y}_t = \mathbf{q} + \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \bar{X}_t \quad (58)$$

where

$$\begin{bmatrix} Q_1 & Q_2 \end{bmatrix} = \begin{bmatrix} -\bar{B}_1 \\ \vdots \\ -\frac{1}{n}\bar{B}_n \end{bmatrix} + \begin{bmatrix} \mathbf{0} & I & \mathbf{0} \end{bmatrix}.$$

This completes the description of the model.

### 3. THE DYNAMICS OF THE ESTIMATED MODEL

The implications of letting the central bank extract information from the term structure depend on the magnitude of the noise in the bond market. There is little information in the term structure when bond prices are very noisy, and including it in the information set of the central bank then has little effect on the dynamics of inflation and output. It is therefore of interest to quantify the variances in the model. The parameters of the model are estimated by Bayesian methods using quarterly data for the US ranging from 1982:Q1 to 2005:Q4. The first 8 observations were used as a convergence sample for the Kalman filter.

The interest rates included are the Federal Funds rate and secondary market rates for 6 and 12 month Treasury Bills. Non-farm real GDP and CPI (excluding food and energy) are used as a measure of output and to calculate quarterly inflation rates. The GDP data series were detrended using the Hodrick-Prescott filter and the CPI and interest rate series were demeaned. By using demeaned data, we do not fully exploit all the information in the yield curve about the structural parameters of the model. However, estimating the model in level form to also match the average slope of the yield curve in US data led to large reductions in overall fit.

The priors on the structural parameters are reported in Table 1. We impose rather tight priors on the degree of price stickiness  $\theta$  which can be measured directly, and the discount factor  $\beta$  which can be backed out from average real interest rates. The priors on the rest of the behavioral parameters are more dispersed. The prior distributions of the variances of the structural shocks of the model are uniform over the interval  $[0, \infty)$  and the prior distributions of the variances of the measurement errors are uniform over the interval  $[0, \sigma_{z_i}^2)$  where  $\sigma_{z_i}^2$  is the variance of the corresponding observable time series.

[Table 1 here.]

All yields except for the short interest rate are assumed to be affected by serially correlated measurement errors/non-fundamental shocks. The variance of the non-fundamental shocks and their persistence is estimated together with the other parameters of the model. We estimate two sets of measurement errors on output and inflation. The first, which we may call the econometric measurement errors, are motivated by the fact that the output and inflation measures that we use do not correspond exactly to the model's concept of output

and inflation as well as that data on GDP may include errors introduced through the data collection process. The variance of the econometric measurement errors are denoted  $\sigma_{vy}^2$  and  $\sigma_{v\pi}^2$ .

We do not want to impose that the time series that we use to estimate the model corresponds exactly to the information that enters into the filtering problem of the central bank. We therefore allow for a second set of measurement errors on the lagged output and inflation measure that the model's central bank observes. These should be interpreted more generally as the central bank's misperceptions of the lagged aggregate variables and their variances are denoted by  $\sigma_{vycb}^2$  and  $\sigma_{v\pi cb}^2$ . The estimated posterior variances of both the econometric measurement errors and the measurement errors of the central banks' observation of lagged output and inflation are reported in Table 1 together with the posterior estimates of the other structural parameters of the model presented in Section 2.

Our primary interest lies in quantifying the variances of the non-fundamental shocks to the term structure since it is these parameters that govern how informative the term structure is as a measure of the current state of the business cycle. Before turning to the consequences for monetary policy of a more or less noisy term structure, we briefly note that the posterior estimates of the parameters governing the behaviour of firms and households are reasonable, and that the standard deviations of the posterior estimates are less dispersed than the priors which indicates that the data is informative about the parameters of the model.

### 3.1. Policy Responses to Shocks and the Role of Term Structure Information.

As argued in the introduction above, one of the main advantages from a policy perspective of using the information in the term structure is that it is observable everyday that the market is open, while macro data takes time to collect. However, for the timeliness of the term structure observations to matter, the term structure cannot be too noisy. If there is useful information in the term structure, one way that this will manifest itself in the estimated model is through the response of the short policy rate to shocks. If there is useful information in the term structure, the short policy rate will respond immediately to shocks, but not otherwise, as the central bank then has to wait for the lagged observation of inflation and output for an indication that a shock has occurred. Figure 1 below shows the impulse responses of output (solid line), inflation (dashed line) and the short interest rate (dotted line) to productivity, demand and cost push shocks at the estimated posterior mode of the model. The short rate responds immediately in the impact period to demand and cost push shocks, but only gradually to a productivity shock. This suggests that there is useful information in the terms structure, at least about demand and cost push shocks.

[Figure 1 here.]

As a comparison, it is interesting to contrast the estimated responses with how the central bank would respond to shocks if it knew the state of the economy perfectly as well as how it would respond to shocks if the term structure was very noisy. These two cases are illustrated in Figure 2 and 3, respectively.

[Figure 2 here.]

The full information policy responses in Figure 2 are qualitatively similar to the estimated responses for demand and cost push shocks. However, with full information, the central bank

responds to a productivity shock by lowering the short interest rate. The lack of immediate response to the productivity shock in the estimated model is thus due to information imperfections on behalf of the central bank, and not because it is not optimal to respond to productivity shocks.

**[Figure 3 here.]**

Figure 3 displays the result of increasing the variance of the non-fundamental shocks in the term structure ten-fold while keeping all other parameters fixed and recomputing the same impulse responses as in Figure 1. We can see that without the information in the term structure, the short interest rate does not respond to any of shocks in the impact period. Instead, policy is only able to respond when the lagged macro variables become observable in the second period after the shock.<sup>5</sup>

The model can also be used to quantify how the unconditional variance of inflation and the output gap (defined as  $y_t - \bar{y}_t$ ) changes when the central bank use the information in the term structure. This exercise suggest that when the central bank uses the information in the term structure the variance of both of these variables are only approximately 2/3 of what it would be otherwise.

The analysis above emphasizes the beneficial policy aspects of responding to movements in the term structure, but since the term structure is noisy this means that sometimes the central bank will inadvertently respond to non-fundamental shocks. The responses of output, inflation and the short interest rate to a unit non-fundamental shock to the 6 and 12 month bond rate is plotted in Figure 4.

**[Figure 4 here.]**

In the absence of noise in the term structure, an increase in either the 6 or 12 month yield signals that a shock has occurred that the market believes that the central bank will respond to by raising interest rates. This is also how the central bank of the model partly interpret a non-fundamental shock to the term structure. A positive noise shock leads the central bank to believe that raising the short interest rate is appropriate, which in turn leads to a fall in both output and inflation. The response is stronger to a non-fundamental shock to the 6 month rate than to the 12 month rate since historically, the variance of the noise has been lower in the 6 month rate than in the 12 month rate. This fact leads the Federal Reserve to attribute a larger fraction of what it observes in the 6 month rate than in the 12 month rate to fundamental sources rather than noise.

That the central bank responds to a non-fundamental increase in long rates by raising short rates does not imply that the model permits self fulfilling expectation shocks. The reason is twofold. Firstly, since the term structure is known by the central bank to be noisy, policymakers will not put enough weight on the observation of bond yields to respond so strongly that a unit increase in a long bond rate justifies it self through the expectations hypothesis. Secondly, the initial increase of the short rate will lower output and inflation.

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<sup>5</sup>The working paper version of this paper, available at [www.rba.gov.au](http://www.rba.gov.au), also includes results using Australian data. The Australian yield curve appears to be too noisy to allow the Reserve Bank of Australia to use it as a timely source of information about the state of the economy, and in the estimated impulse responses the short rate do not respond immediately to shocks, i.e. the Australian case is much closer to Figure 3 than to Figure 1.

Ceteris paribus, this means that in the next period when the output and inflation outcomes of the impact period are observed, the central bank will want to lower interest rates. These two forces rule out self fulfilling expectations shocks to longer maturity yields.

**3.2. Variance Decomposition.** The estimated model can be used to decompose historical variances of the endogenous variables into their exogenous sources. Table 2 reports the unconditional variance decomposition evaluated at the posterior modes as well as the 95 % probability intervals. Output variance is primarily caused by productivity shocks which also explain more than half of the variance of inflation. Demand shocks appear to not be very important for output variance, but explain more than half the variance of the short interest rate which suggest that the Federal Reserve has successfully raised interest rates to counter exogenous demand pressure during the sample period. The posterior mode estimates suggest that cost push shocks are not a major source of macroeconomic variability.

[Table 2 here.]

Perhaps the more interesting part of Table 2 are the four columns to the right. These columns report the proportion of the variances of output, inflation and interest rates of different maturities that are caused by non-structural shocks, i.e. by central bank misperceptions about lagged output and inflation and the non-fundamental shocks to the term structure. The channel through which these shocks affect the macro economy is the short interest rate. In the present model, all non-systematic changes in monetary policy are attributed to misperceptions of the state of the economy by the central bank. The four non-structural shocks thus provide a way to interpret what in a full information model would be termed "monetary policy shocks".

The model attributes policy shocks to come mostly either from misperceptions about lagged output, which explains 11% of short interest rate variability, or from non-fundamental shocks to the 6 month yields, which explain 17% of the variability short interest rates. Through their effect on the short rate, these shocks also affect the real economy and misperceptions about output particularly, explain a substantial fraction of inflation and output variability (7% and 30% respectively).

This magnitude of interest rate variance that is attributable to "policy shocks" is comparable to what full information studies have found, e.g. Smets and Wouters (2004). That a large portion of the "policy shocks" comes from the bond market suggest that there may have been episodes where misperceptions about the state of the economy may have been shared by the bond market participants. These non-fundamental shocks feed into output and inflation through the short term interest rate, and it may appear as if responding to these shocks is not optimal. However, since the central bank is assumed to respond with statistically optimal weights to movements in the term structure, the benefits of having more accurate estimates of the state on average outweighs the cost of occasionally responding to "false alarms".

**3.3. The (lack of) information in longer maturity yields.** In the variance decomposition tables, we could see that the fraction of the variance of yields that is explained by non-fundamental shocks is larger for the 1 year bonds than for 6 month bonds and almost all information in the term structure about the state of the economy is in the observation

of the 6 month yield. Excluding the 1 year yield hardly changes the responses to shocks of output, inflation and short interest rates at all. The model was also estimated including 2 year and 5 year yields in the vector of variables that the central bank can observe. These longer yields held virtually no information about the state and their variance was almost entirely explained by non-fundamental shocks. That longer maturity bonds are uninformative about the state of the economy is the dual of the fact that standard macro models are not very good at explaining the movements of long maturity yields (see for instance Gurkaynak, Sack and Swanson (2005)). (This is also true for theoretically less stringent models like Ang and Piazzesi's (2003) who find that macro factors contribute most explanatory power to the yield curve at short-to-medium maturities.) If the business cycle model is not very good at explaining long yields, then movements in long yields will not be very informative about the current state of the business cycle. This is not to say that there is no correlation between long yields and the business cycle, but rather that if we want to interpret this correlation using structural models that can guide policy, then we need to develop a better understanding of what drives long maturity yields.

**3.4. Robustness.** In the baseline estimation above it was assumed that the central banks had an explicit interest rate smoothing objective. An alternative explanation of inertial interest rate changes could be that central banks move slowly because they want to accumulate more information before they move. The model was re-estimated after imposing no interest rate smoothing, that is, by setting  $\lambda_i = 0$ . This lead to reductions in the marginal likelihoods and the corresponding posterior odds ratios do not support the hypothesis that  $\lambda_i = 0$ .<sup>6</sup>

In the estimation, it was also imposed that the Federal Reserve used the information in the term structure to set policy. If this assumption is incorrect, it would bias the estimates of the noise in the term structure upwards. The log likelihood decreases when we re-estimate the model without assuming that the central banks use the information in the term structure. Though the change in marginal likelihood is small, the direction of the change lends some empirical support for the assumption that the Fed uses the information in the term structure to set policy, in addition to anecdotal evidence in the form of quotes from Federal Reserve officials cited in the introduction.

#### 4. CONCLUSIONS

This paper has presented a general equilibrium model of monetary policy where the central bank operates in an uncertain environment and uses information contained in the term structure to estimate the underlying state of the economy more efficiently. This set up creates a link between the term structure and the macro economy that is novel to the literature. A movement in the term structure signals that a change in the short term interest rate set by the central bank may be desirable, which when implemented in turn affects aggregate demand.

Söderlind and Svensson (1997) warn that “central banks should not react mechanically to [market expectations]” since this may lead to a situation of “the central bank chasing the

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<sup>6</sup>More details on posterior mode estimates and likelihoods for the alternative specifications are available from the author upon request.

market, and the market simultaneously chasing the central bank". This argument is formalized in Bernanke and Woodford (1997) where the authors show that if central banks react to market expectations, a situation with a multiplicity of equilibria or where no equilibrium exists may arise. In this paper we have argued that there may be benefits from systematic reactions to market expectations, but with some important qualifications. The non-existence of equilibria arises in the model of Bernanke and Woodford because the central bank can extract the underlying state perfectly from observing the expectations of the private sector. Inflation will thus always be on target. But if inflation is always on target and private agents only care about accurate inflation forecasts there is no incentive for the private sector to pay a cost to be informed about the underlying shock, and observing expectations will not reveal any information. The model here differs because, to the extent that there is noise in the bond market, the central bank cannot extract the underlying shock perfectly. Thus there will always exist a cost of information gathering that is small enough to make it profitable for the private sector to acquire information about the underlying shock, even if private agents only cared about having accurate inflation forecasts. Additionally, in this model the forecasting problem of private agents involves more than accurately forecasting inflation since bond prices depend on real factors through the stochastic discount factor as well as the price level. In so far as the real discount factor is affected by the underlying state, agents will have an incentive to collect information about it, regardless of the behavior of inflation.

Ultimately, the informational content of the term structure is an empirical question. The model presented here provides a coherent framework within which any information about the state of the economy that is contained in the term structure can be quantified in a general equilibrium setting. The model explicitly takes into account that the central bank may use the information in the term structure to set policy and therefore influences what it observes. The model was estimated on US data using Bayesian methods. The empirical exercise suggests that there is some information in the US term structure that allow the Federal Reserve to respond to shocks in a timely manner.

Most of the information in the US term structure about the state of the business cycle could be found in yields with maturities of less than one year. That longer maturity bonds are uninformative about the state of the economy is a consequence of the fact that the standard macro models are not very good at explaining movements in long maturity yields (see for instance Gurkaynak, Sack and Swanson (2005)). If a business cycle model is not very good at explaining long yields, then movements in long yields will not be very informative about the current state of the business cycle as defined by the model at hand. There is reduced form evidence that long maturity yields are correlated with the business cycle, with perhaps the most famous link being between an inverted yield curve and the onset of a recession (Harvey (1993)) which suggest that there may be potentially useful information also in the long end of the term structure. However, the link between long term interest rates and optimizing behaviour by individual investors are still quite poorly understood. Until a better understanding of what drives long rates is achieved, we may have to settle for looking at the short end of the yield curve to find clues about the current state of the business cycle that we can interpret using structural micro founded models.

## REFERENCES

- [1] **Amato, J. and T. Laubach (2004)**, “Implications of habit formation for optimal monetary policy”, *Journal of Monetary Economics* Vol 51, pp 305-325.
- [2] **Ang, A. and M. Piazzesi (2003)**, “A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables”, *Journal of Monetary Economics*, 50, pp.745-787.
- [3] **Ang, A., M. Piazzesi and M. Wei (2006)**, “What does the yield curve tell us about GDP Growth?”, *Journal of Econometrics*, 131, pp359-403.
- [4] **Bekaert, G., S. Cho and A. Moreno (2005)**, “New-Keynesian Macroeconomics and the Term Structure”, NBER Working Paper 11340
- [5] **Bernanke, B. (2004)**, “What Policy Makers Can Learn from Asset Prices”, Remarks before *The Investment Analysts Society of Chicago*, Chicago, Illinois, April 15th available at <http://www.federalreserve.gov/BoardDocs/Speeches/2004/20040415/default.htm>.
- [6] **Bernanke, B. and M. Woodford (1997)**, Inflation Forecasts and Monetary Policy”, *Journal of Credit Money and Banking*, vol. 29, No. 4 (Part 2).
- [7] **Campbell, J. Y., and J. H. Cochrane (1999)**, “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107(2), pp 205–251.
- [8] **Christiano, L.J., M. Eichenbaum and C. Evans (2001)**, “Nominal rigidities and the dynamic effects of a shock to monetary policy”, Working Paper 0107, Federal Reserve Bank of Cleveland.
- [9] **Ellingsen, T and U. Söderström (2001)**, “Monetary Policy and Market Interest Rates”, *The American Economic Review*, Vol. 91, No. 5., pp. 1594-1607.
- [10] **Estrella, A. and F. Mishkin (1998)**, “Predicting U.S. Recessions: Financial Variables as leading indicators”, *Review of Economics and Statistics* vol. 80, pp 45-61.
- [11] **Goodfriend, M., (1998)** “Using the Term Structure of Interest Rates for Monetary Policy,” *Economic Quarterly*, Federal Reserve Bank of Richmond, Summer Issue, pages 13-30.
- [12] **Gürkaynak, R.S., B. Sack and E. Swanson, (2005)**, ”The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models”, *American Economic Review*, vol 95, pp 425-436.
- [13] **Hayek, F. A. (1945)**, “The Use of Knowledge in Society, ”, *American Economic Review*, 1945, 35 (4), 519—530.
- [14] **Hansen, L.P. and T.J. Sargent (2004)**, “Recursive Models of Dynamic Linear Economies”, unpublished book manuscript New York University and University of Chicago.
- [15] **Harvey, C.R. (1988)**, “The Real Term Structure and Consumption Growth”, *Journal of Financial Economics* vol 22.
- [16] **Harvey, C.R. (1993)**, ”The term structure and forecasts of economic growth”, *Financial Analyst Journal*, vol 49, pp6-8;
- [17] **Hördahl, P., O. Tristani, and D. Vestin (2004)**, “A Joint Econometric Model of Macroeconomic and Term Structure Dynamics”, forthcoming in the *Journal of Econometrics*.
- [18] **Mishkin, F. S. (1990)**, “The Information in the Longer maturity term Structure about Future Inflation”, *Quarterly Journal of Economics*, vol 55, pp 815-828.
- [19] **Smets, F. and R. Wouters (2004)**, “Comparing Shocks and Frictions in the U.S. and in the Euro Area Business Cycles: A Bayesian DSGE Approach”, *ECB working paper 391*.
- [20] **Söderlind, P. and L.E.O. Svensson (1997)**, “New techniques to extract market expectations from financial instruments”, *Journal of Monetary Economics*, 40, pp 383-429.
- [21] **Söderlind, P. (1999)**, “Solution and Estimation of Rational Expectations Macromodels with Optimal Policy”, *European Economic Review*, 43, pp813-823.
- [22] **Svensson, L.E.O., and Woodford, M. (2004)**, “Indicator variables for optimal policy under asymmetric information”, *Journal of Economic Dynamics and Control*, vol 28, Issue 4, pp 661-690.
- [23] **Wu, T. (2002)**, “Macro Factors and the Affine Term Structure of Interest Rates ”, *Federal Reserve Bank of San Francisco Working paper #2002-06*.

## APPENDIX A. THE MODEL

The model can be put in compact form

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = A \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + B i_t + C \varepsilon_t \quad (59)$$

$$X_{1,t} = [a_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, i_{t-1}, \Delta i_t, \mathbf{v}_t]' \quad (60)$$

$$X_{2,t} = [y_t, \pi_t]' \quad (61)$$

$$\varepsilon_t = [\varepsilon_t^a, \varepsilon_t^y, \varepsilon_t^\pi, \mathbf{e}_t^y]' \quad (62)$$

where the coefficient matrices  $A$ ,  $B$  and  $C$  are given by

$$A = A_0^{-1} A_1, \quad B = A_0^{-1} B_1, \quad C = A_0^{-1} \begin{bmatrix} C_1 \\ \mathbf{0}_{2 \times 1} \end{bmatrix}$$

$$A_0 = \begin{bmatrix} I_5 & \mathbf{0} & 0 & \mathbf{0} & 0 & 0 & 0 \\ \mathbf{0} & 1 & 0 & \mathbf{0} & 0 & 0 & 0 \\ 0 & -1 & 1 & \mathbf{0} & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I_5 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & \mathbf{0} & \mu_{yf} & \phi & 0 \\ 0 & 0 & 0 & \mathbf{0} & 0 & 0 & \mu_{\pi f} \end{bmatrix} \quad (63)$$

$$A_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \mathbf{0} & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \mathbf{0} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & W & \mathbf{0} & \mathbf{0} \\ 0 & -\mu_{yb} & 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 1 & 0 \\ \kappa \frac{1+\varphi}{(\varphi+\gamma)} & \kappa \frac{\eta(1-\gamma)}{(\varphi+\gamma)} & -\mu_{\pi b} & 0 & 0 & 0 & 0 & \mathbf{0} & -\kappa(\varphi+\gamma) & 1 \end{bmatrix} \quad (64)$$

where

$$\mu_{yf} \equiv \frac{\gamma}{\gamma - \eta + \gamma\eta}, \quad \mu_{yb} \equiv \frac{-\eta(1-\gamma)}{\gamma - \eta + \gamma\eta}$$

$$\phi \equiv \frac{1}{\gamma - \eta + \gamma\eta}$$

$$\mu_{\pi f} \equiv \frac{\beta\theta}{\theta + \omega(1 - \theta(1 - \beta))}, \quad \mu_{\pi b} \equiv \frac{\omega}{\theta + \omega(1 - \theta(1 - \beta))}$$

$$\kappa \equiv \frac{(1 - \omega)(1 - \theta)(1 - \theta\beta)}{\theta + \omega(1 - \theta(1 - \beta))}$$

$$W = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \rho\gamma_2 & 0 \\ \mathbf{0} & 0 & \rho\gamma_4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \mathbf{0}_{5 \times 1} \\ 1 \\ \mathbf{0}_{6 \times 1} \\ \phi \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} \end{bmatrix} \quad (65)$$

$$\mathcal{L}_t = E_t \left[ \sum_{k=0}^{\infty} \beta^k [\lambda_y (y_{t+k} - \bar{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_t - i_{t-1})^2] \right] \quad (66)$$

## APPENDIX B. THE LIKELIHOOD FUNCTION

Form a state space system of the AR(1) process of the state  $\bar{X}_t$

$$\bar{X}_t = M\bar{X}_{t-1} + N\varepsilon_t \quad (67)$$

$$\hat{Z}_t = \hat{\mu} + D\bar{X}_t + \hat{v}_t \quad (68)$$

$$E\hat{v}_t\hat{v}_t' = \hat{\Sigma}_{vv} \quad (69)$$

where  $\hat{Z}_t$  is the vector of variables that are observable and  $\hat{\Sigma}_{vv}$  is the covariance matrix of the measurement errors given by

$$\hat{\Sigma}_{vv} = \begin{bmatrix} \sigma_{vy}^2 & 0 & \mathbf{0} \\ 0 & \sigma_{v\pi}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (70)$$

Construct the innovation series  $\{u_t\}_{t=0}^T$  from the innovation representation

$$\hat{X}_{t+1} = \hat{\mu} + M\hat{X}_t + M\hat{K}u_t \quad (71)$$

$$\hat{Z}_t = D\hat{X}_t + u_t \quad (72)$$

by rearranging to

$$u_t = \hat{Z}_t - D\hat{X}_t \quad (73)$$

where  $K$  is the Kalman gain matrix

$$\hat{K} = \hat{P}D'(D\hat{P}D' + \hat{\Sigma}_{vv})^{-1} \quad (74)$$

$$\hat{P} = M(\hat{P} - \hat{P}D'(D\hat{P}D' + \hat{\Sigma}_{vv})^{-1}D\hat{P})M' + N\Sigma N'. \quad (75)$$

The log likelihood  $\mathcal{L}(\hat{Z} | \Theta)$  of observing the data  $\hat{Z}$  for a given set of parameters  $\Theta$  can then be computed as

$$\mathcal{L}(\hat{Z} | \Theta) = -.5 \sum_{t=0}^T [p \ln(2\pi) + \ln |\Omega| + u_t' \Omega^{-1} u_t] \quad (76)$$

where

$$\Omega = M\hat{P}M' + \hat{\Sigma}_{vv}. \quad (77)$$

The posterior mode  $\hat{\Theta}$  is then given by

$$\hat{\Theta} = \arg \max [\mathcal{L}(\Theta) + \mathcal{L}(\hat{Z} | \Theta)] \quad (78)$$

where  $\mathcal{L}(\Theta)$  denotes the log of the prior likelihood of the parameters  $\Theta$ . The posterior mode was found using Bill Goffe's simulated annealing minimizer (available at <http://cook.rfe.org/>). The posterior standard deviations were calculated using Gary Koop's Random Walk Metropolis-Hastings distribution simulator (available at <http://www.wiley.co.uk/koopbayesian/>).

**Table 1**  
**Prior and estimated posteriors of structural parameters**

	Prior mode	Prior s.e.	Distribution	Posterior	s.e.
$\gamma$	2	0.32	normal	1.82	0.22
$\varphi$	3	1	normal	0.03	0.18
$\eta$	1	0.22	normal	1.19	0.08
$\theta$	0.75	0.05	beta	0.81	0.04
$\omega$	0.3	0.05	beta	0.37	0.03
$\beta$	0.99	0.03	beta	0.98	0.02
$\rho$	0.5	0.15	beta	0.86	0.11
$\lambda_y$	0.5	0.22	normal	0.52	0.09
$\lambda_i$	0.5	0.22	normal	0.53	0.13
$\sigma_a$	-	-	uniform $(0, \infty)$	$2.04 \times 10^{-2}$	$1.2 \times 10^{-4}$
$\sigma_y$	-	-	uniform $(0, \infty)$	$2.79 \times 10^{-3}$	$1.1 \times 10^{-5}$
$\sigma_\pi$	-	-	uniform $(0, \infty)$	$1.66 \times 10^{-4}$	$4.5 \times 10^{-5}$
$\sigma_{vycb}$	-	-	uniform $(0, \sigma_{zy})$	$9.64 \times 10^{-3}$	$4.2 \times 10^{-8}$
$\sigma_{v\pi cb}$	-	-	uniform $(0, \sigma_{z\pi})$	$7.69 \times 10^{-6}$	$2.2 \times 10^{-5}$
$\sigma_{vy}$	-	-	uniform $(0, \sigma_{zy})$	$1.69 \times 10^{-6}$	$3.3 \times 10^{-6}$
$\sigma_{v\pi}$	-	-	uniform $(0, \sigma_{z\pi})$	$1.24 \times 10^{-6}$	$1.4 \times 10^{-5}$
$\sigma_{y^2}$	-	-	uniform $(0, \sigma_{zy^2})$	$7.15 \times 10^{-4}$	$5.6 \times 10^{-6}$
$\sigma_{y^4}$	-	-	uniform $(0, \sigma_{zy^4})$	$1.02 \times 10^{-3}$	$1.1 \times 10^{-5}$
$\rho_{y^2}$	-	-	uniform $(0, 1)$	0.74	0.05
$\rho_{y^4}$	-	-	uniform $(0, 1)$	0.83	0.05
$\mathcal{L}(\hat{\Theta})$	3.8				
$\mathcal{L}(Z   \hat{\Theta})$	2272.9				
$\mathcal{L}(\hat{\Theta}   Z)$	2276.7				

**Table 2 Variance Decomposition  
(2.5%-97.5% probability intervals)**

	$\varepsilon_t^a$	$\varepsilon_t^y$	$\varepsilon_t^\pi$	$v_t^{ycb}$	$v_t^{\pi cb}$	$v_t^{\mathcal{Y}^2}$	$v_t^{\mathcal{Y}^4}$
$y_t$	0.79 (0.69–0.84)	0.12 (0.08–0.20)	0.01 (0–0.04)	0.07 (0.04–0.08)	0 (0–0)	0.01 (0.01–0.02)	0 (0–0)
$\pi_t$	0.63 (0.54–0.71)	0.05 (0.01–0.12)	0.01 (0–0.04)	0.30 (0.15–0.38)	0 (0–0)	0.01 (0–0.02)	0 (0–0)
$i_t$	0.17 (0.09–0.24)	0.51 (0.41–0.54)	0.05 (0–0.12)	0.11 (0.05–0.17)	0 (0–0.04)	0.17 (0.13–0.21)	0 (0–0)
$\mathcal{Y}_t^2$	0.15 (0.07–0.23)	0.38 (0.31–0.43)	0.03 (0–0.06)	0.06 (0.03–0.09)	0 (0–0.01)	0.39 (0.30–0.48)	0 (0–0)
$\mathcal{Y}_t^4$	0.03 (0.02–0.07)	0.07 (0.05–0.11)	0 (0–0.02)	0.01 (0.01–0.02)	0 (0–0)	0.01 (0.01–0.02)	0.87 (0.79–0.89)

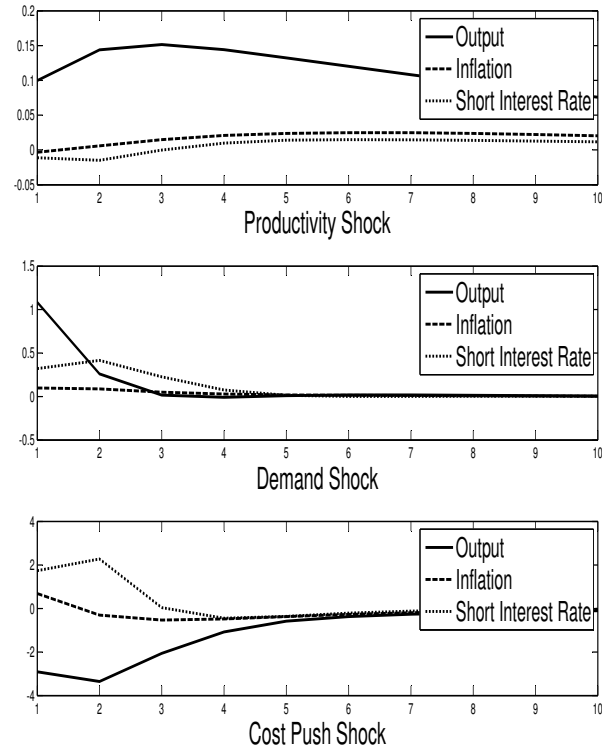


FIGURE 1. Estimated impulse response to technology, demand and cost push shocks

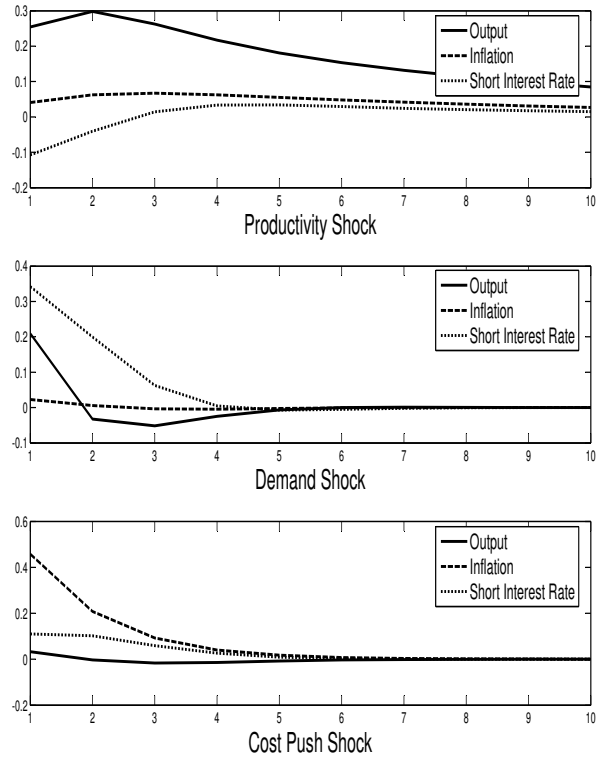


FIGURE 2. Impulse response with perfectly informed central bank

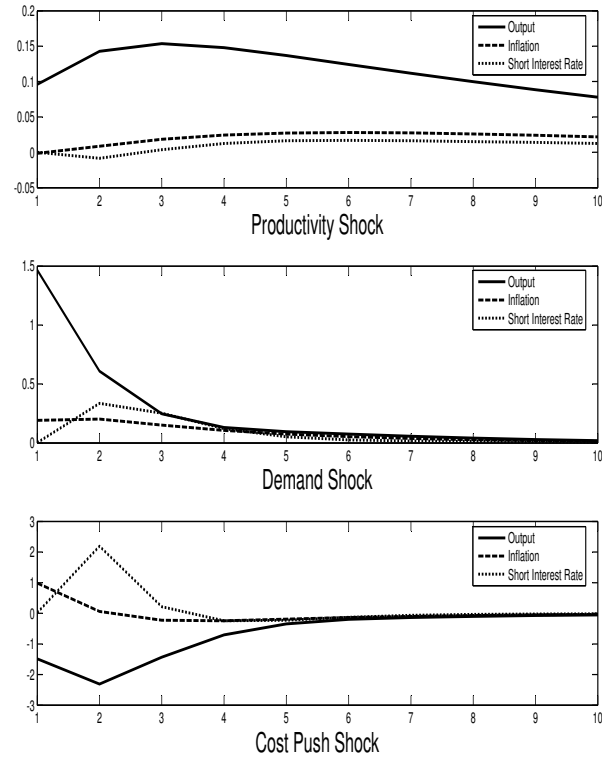


FIGURE 3. Impulse response without term structure information

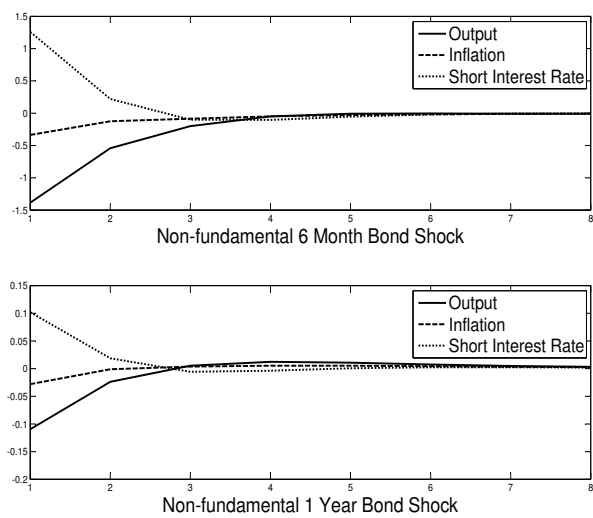


FIGURE 4. Impulse responses to non fundamental shock to 6 month bond rate