SIGNALS, BELIEFS AND UNUSUAL EVENTS

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These notes presents and discusses ideas related to the modeling framework in my paper *Man-bites-dog Business Cycles* (Nimark 2012). The notes, just like the paper, is concerned with modeling a particular aspect of news reporting, namely that more unusual events are considered more newsworthy than more commonplace events. The main purpose of the notes, relative to the paper, is to provide a more step-by-step derivation of how to use the techniques in the paper. The hope is this that the notes will help a person who may want to apply similar techniques to a different question. The notes also provide more examples of information structures that are not in the paper for space constraints reasons.

The background for the formal techniques introduced below is that to take optimal actions, agents need information. Conceptually, information about the current state of the world can be divided into at least three categories. What we may call *local information* is information that agents observe directly through their interactions in markets, e.g. through buying and selling goods or through participating in the labor market. A second type of information is what we may call *statistics.* Statistics are collected and summarized by (often government) organizations and made available to a broader public through web sites and printed media. Statistics are normally reported regardless of the realized values of the variable that they refer to and often according to a pre-specified schedule. A third type is information provided by the news media, such as newspapers and television programs. News media may be the main source of information for a large section of the general population and one service that

Date: January 24, 2013.

the news media provides is to select what events to report. This editorial function of the news media is necessary since it is simply not possible for a newspaper or a television news program to report all events that has occurred on a given day or during a given week.

So how do newspapers decide what to report? One dimension that arguably makes an event more newsworthy is how unusual it is. This is captured by the expression that "dog-bites-man is *not* news, but man-bites-dog *is* news". This is from the Wikipedia entry http://en.wikipedia.org/wiki/Man_bites_dog_(journalism):

"*The phrase man bites dog is a shortened version of an aphorism in journalism which describes how an unusual, infrequent event is more likely to be reported as news than an ordinary, everyday occurrence with similar consequences, such as a dog biting a person ("dog bites man"). An event is usually considered more newsworthy if there is something unusual about it; a commonplace event is less likely to be seen as newsworthy"*

A prime example of man-bites-dog news reporting is the *Movers* segment on Bloomberg Television. In a typical segment, the price movements of a few stocks are reported along with short statements on the probable causes of these movements. The stocks in question are a small sub-sample of all stocks traded and are selected on the basis of having had the largest price movements during the day. Unusually large price movements are thus more likely to be reported than more common price movements. Because of the way that stock prices are selected for inclusion in the *Movers* segment, the variance of a stock's price conditional on it being mentioned in the *Movers* segment is thus clearly larger than its unconditional variance. That is, the availability of the signal about the stocks price is in itself informative about the distribution of the stock price.

Man-bites-dog signals become more interesting when they are not perfectly precise (i.e. when the signals are noisy). These notes describe formal methods to analyze the implications of this aspect of news reporting for economic agents' beliefs and decisions.

1. Signals and unusual events

This section introduces the concept of man-bites-dog signals formally. A signal y about a latent variable x will be called a man-bites-dog signal if it is more likely to be available when the realization of x is more unusual (in a sense that will be made precise below). When the availability of the signal y depends on the realized value of x, Bayes' rule implies that it is *as if* the latent variable x is drawn from a different distribution when the signal y is available compared to when it is not. In particular, conditional on the signal y being available, the relative probability of unusual events increases. This section draws out the implications of this fact for how Bayesian agents update their beliefs in response to man-bites-dog signals.

1.1. **Signal availability and conditional distributions.** We will denote the unconditional probability density function of the latent variable of interest x as $p(x)$. An unusual realization of x is thus a realization for which $p(x)$ is small. We are interested in information structures in which the probability of observing the signal y about x is larger for relatively unusual realizations of x. To help distinguish between a particular realization of the signal y and the event that the signal y is available, the indicator variable S is defined to take the value 1 when the signal y about x is available and 0 otherwise. We can then define a man-bites-dog signal as follows.

Definition 1. *The signal y is said to be a man-bites-dog signal if for any two realizations of x denoted x' and x'' such that*

$$p(x') < p(x'') \tag{1.1}$$

the inequality

$$p(S = 1 \mid x') > p(S = 1 \mid x'') \tag{1.2}$$

holds.

The first inequality in the definition simply establishes that x' is a more unusual realization than x''. The second inequality formalizes the notion that a more unusual realization of x is considered more newsworthy than a more common realization. Because the signal y is more likely to be available for some realized values of x than for others, the availability of the signal y is in itself informative about the distribution of x. More specifically, using Bayes' rule, the next proposition shows that conditional on the event that the signal y is available, the probability of more unusual realizations of x increases.

Proposition 1. *The more unusual realization x' is relatively more likely when the signal y is available, i.e.*

$$\frac{p\left(x' \mid S=1\right)}{p\left(x'' \mid S=1\right)} > \frac{p\left(x'\right)}{p\left(x''\right)} \tag{1.3}$$

Proof. Dividing Bayes' rule for conditional probabilities

$$p\left(x \mid S=1\right) p\left(S=1\right) = p\left(S=1 \mid x\right) p\left(x\right) \tag{1.4}$$

for x' by the same expression for x'' gives

$$\frac{p\left(x' \mid S=1\right)}{p\left(x'' \mid S=1\right)} = \frac{p(S=1 \mid x')}{p(S=1 \mid x'')} \frac{p\left(x'\right)}{p\left(x''\right)} \tag{1.5}$$

The proof then follows directly from the fact that the inequality (1.2) in Definition 1 implies that

$$\frac{p(S=1 \mid x')}{p(S=1 \mid x'')} > 1 \tag{1.6}$$

□

Proposition 1 states that the relative probability of observing the more unusual realization x' compared to the more common realization x'' is larger conditional on the signal y being available. The availability of a man-bites-dog signal thus implies redistributing probability mass away from more likely outcomes towards relatively less likely outcomes. By a completely symmetric argument we have

$$\frac{p\left(x' \mid S=0\right)}{p\left(x'' \mid S=0\right)} < \frac{p\left(x'\right)}{p\left(x''\right)} \tag{1.7}$$

so that the absence of a man-bites-dog signal implies that probability mass should be redistributed towards relatively more likely outcomes. Because the availability of the signal y is a discrete event, whether y is available or not thus effectively splits the unconditional distribution $p(x)$ into the two distinct conditional distributions $p(x \mid S = 1)$ and $p(x \mid S = 0)$.

1.2. **Example: A discrete distribution.** To see the implications of the results derived above, consider a simple discrete distribution of x. We want to show that a signal y about the latent variable x is will be interpreted differently if it always available compared to if it is a man-bites-dog signal who's availability depends on the value of x.

Lets say that x can take the value high (H), medium (M) or low (L) with probabilities given by

$$p(x^H) = 1/4 \tag{1.8}$$

$$p(x^M) = 1/2 \tag{1.9}$$

$$p(x^L) = 1/4 \tag{1.10}$$

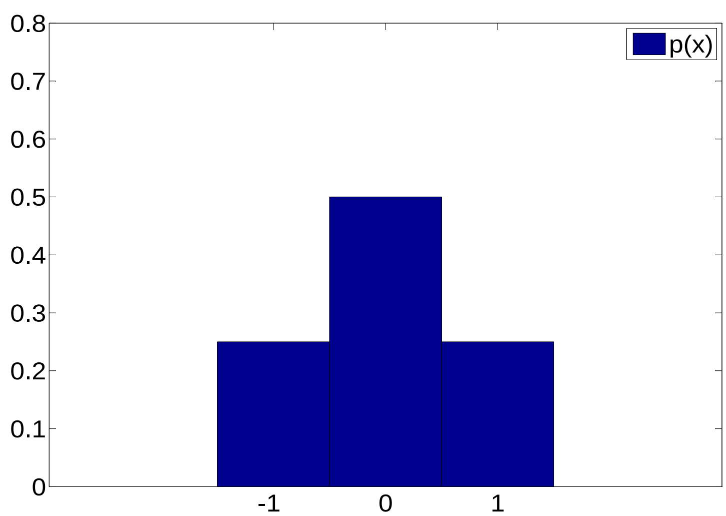

FIGURE 1. Unconditional distribution of x

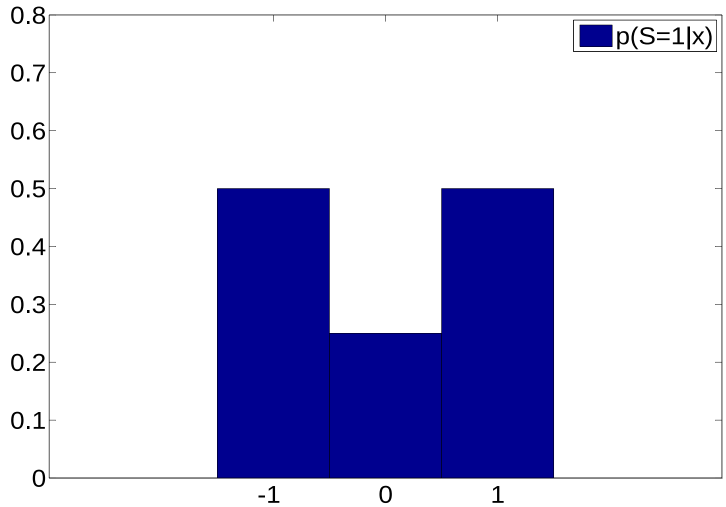

FIGURE 2. The probability of observing y conditional on x

Now let the probability of observing the signal y be given by

$$p(S = 1 \mid x^H) = 1/2 \tag{1.11}$$

$$p(S = 1 \mid x^M) = 1/4 \tag{1.12}$$

$$p(S = 1 \mid x^L) = 1/2 \tag{1.13}$$

Clearly, this is is a man-bites-dog structure according to Definition 1: The probability of observing the signal y is higher when x is high or low than when x takes the medium value. Now apply Bayes' rule

$$p\left(x \mid S = 1\right) = \frac{p\left(S = 1 \mid x\right) p\left(x\right)}{p\left(S = 1\right)} \tag{1.14}$$

to compute the probability of each state of x conditional on the signal y being available

$$\begin{aligned} p\left(x^H \mid S = 1\right) &= \frac{1/2 \times 1/4}{1/4 \times 1/2 + 1/2 \times 1/4 + 1/4 \times 1/2} \\ &= 1/3 \end{aligned} \tag{1.15}$$

$$p\left(x^M \mid S=1\right) = \frac{1/4 \times 1/2}{1/4 \times 1/2 + 1/4 \times 1/2 + 1/4 \times 1/2} = 1/3 \tag{1.16}$$

$$p\left(x^L \mid S=1\right) = \frac{1/2 \times 1/4}{1/4 \times 1/2 + 1/2 \times 1/4 + 1/4 \times 1/2} = 1/3 \tag{1.17}$$

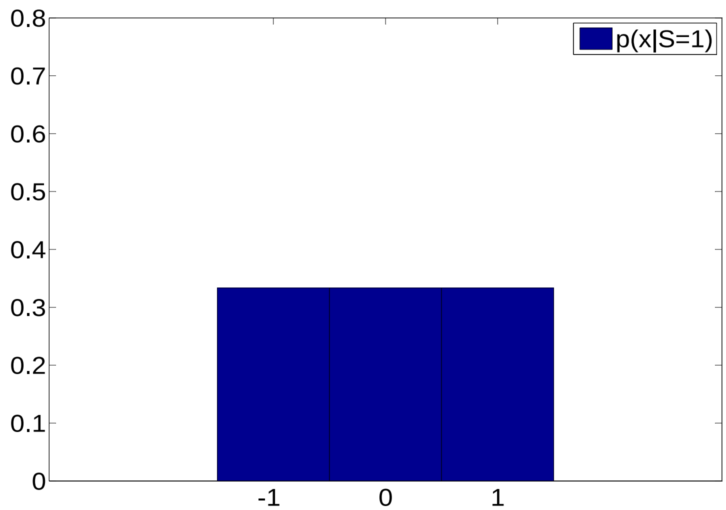

FIGURE 3. Distribution of x conditional y being available.

We can see that conditioning on the availability of the signal y implies redistributing probability mass from the more common realization x^M towards the less common realizations x^H and x^L. We thus have that

$$\frac{p\left(x^H \mid S=1\right)}{p\left(x^M \mid S=1\right)} > \frac{p(x^H)}{p(x^M)} \tag{1.18}$$

and

$$\frac{p\left(x^L \mid S=1\right)}{p\left(x^M \mid S=1\right)} > \frac{p(x^L)}{p(x^M)} \tag{1.19}$$

since

$$1 = \frac{1/3}{1/3} > \frac{1/4}{1/2} = 1/2 \tag{1.20}$$

So far, we have only conditioned on the availability of the signal y. Naturally, to form a posterior belief agents will also condition on the contents of the signal y. Now define the different realizations of the signal y (when available) to have the following probabilities

$$\begin{aligned} p\left(y^H \mid x^H\right) &= 3/4 \\ p\left(y^M \mid x^H\right) &= 1/4 \\ p\left(y^L \mid x^H\right) &= 0 \end{aligned} \tag{1.21}$$

$$\begin{aligned} p\left(y^H \mid x^M\right) &= 1/4 \\ p\left(y^M \mid x^M\right) &= 1/2 \\ p\left(y^L \mid x^M\right) &= 1/4 \end{aligned} \tag{1.22}$$

$$\begin{aligned} p\left(y^H \mid x^L\right) &= 0 \\ p\left(y^M \mid x^L\right) &= 1/4 \\ p\left(y^L \mid x^L\right) &= 3/4 \end{aligned} \tag{1.23}$$

That is, the signal is more likely to be y^H when x is x^H and so on, but the signal is noisy in the sense that the realized value of y may not always correspond to the true realized value of x.

Let's compute the probabilities of the different possible states of x when the signal is high y^H. Using Bayes' Rule again[1] we have that $p\left(x \mid y^H\right)$ is given by

$$\begin{aligned} p\left(x^H \mid y^H\right) &= \frac{p\left(y^H \mid x^H\right) p\left(x^H \mid S=1\right)}{p\left(y^H\right)} && (1.24)\\ &= \frac{3/4 \times 1/3}{3/4 \times 1/3 + 1/4 \times 1/3 + 0 \times 1/3} && (1.25)\\ &= \frac{3/12}{3/12 + 1/12} = 3/4 && (1.26) \end{aligned}$$

$$\begin{aligned} p\left(x^M \mid y^H\right) &= \frac{p\left(y^H \mid x^M\right) p\left(x^M \mid S=1\right)}{p\left(y^H\right)} && (1.27)\\ &= \frac{1/4 \times 1/3}{3/4 \times 1/3 + 1/4 \times 1/3 + 0 \times 1/3} && (1.28)\\ &= 1/4 && (1.29) \end{aligned}$$

$$\begin{aligned} p\left(x^L \mid y^H\right) &= \frac{p\left(y^H \mid x^L\right) p\left(x^L \mid S=1\right)}{p\left(y^H\right)} && (1.30)\\ &= 0 && (1.31) \end{aligned}$$

Now, let's compare these numbers to what these probabilities would be if the signal y was always available so that $p(x) = p(x \mid S=1)$:

$$\begin{aligned} p\left(x^H \mid y^H\right) &= \frac{p\left(y^H \mid x^H\right) p\left(x^H\right)}{p\left(y^H\right)} && (1.32)\\ &= \frac{3/4 \times 1/4}{3/4 \times 1/4 + 1/4 \times 1/2 + 0 \times 1/4} && (1.33)\\ &= 3/5 < 3/4 && (1.34) \end{aligned}$$

[1] And not for the last time.

$$p\left(x^M \mid y^H\right) = \frac{p\left(y^H \mid x^M\right) p\left(x^M\right)}{p\left(y^H\right)} \tag{1.35}$$

$$= \frac{1/4 \times 1/2}{3/4 \times 1/4 + 1/4 \times 1/2 + 0 \times 1/4} \tag{1.36}$$

$$= 2/5 > 1/4 \tag{1.37}$$

$$p\left(x^L \mid y^H\right) = \frac{p\left(y^H \mid x^L\right) p\left(x^L\right)}{p\left(y^H\right)} \tag{1.38}$$

$$= \frac{0 \times 1/2}{3/4 \times 1/4 + 1/4 \times 1/2 + 0 \times 1/4} \tag{1.39}$$

$$= 0 \tag{1.40}$$

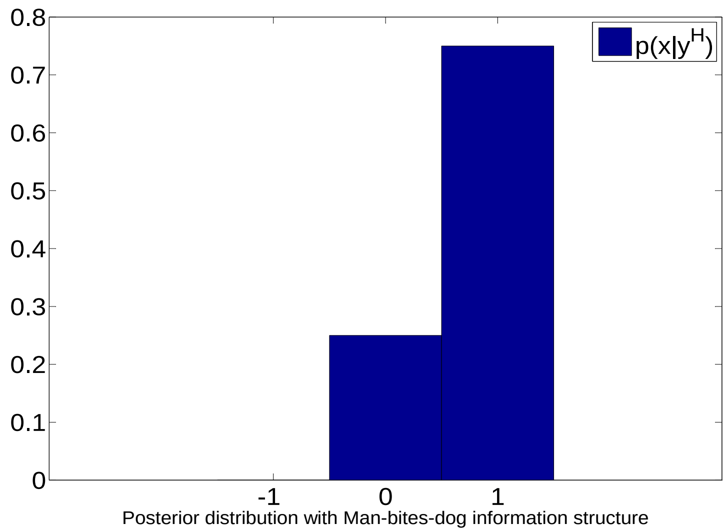

FIGURE 4. Distribution of x conditional on $y = y^H$ when y is more likely to be available when x is high or low.

This demonstrates that when the availability of the signal depends on realized value of the latent variable of interest, that affect how the signal is interpreted. In this case, after conditioning on the signal y, agents put a higher probability on unusual states when y is a man-bites-dog signal compared to the case when the availability of y is independent of the realized value of x.

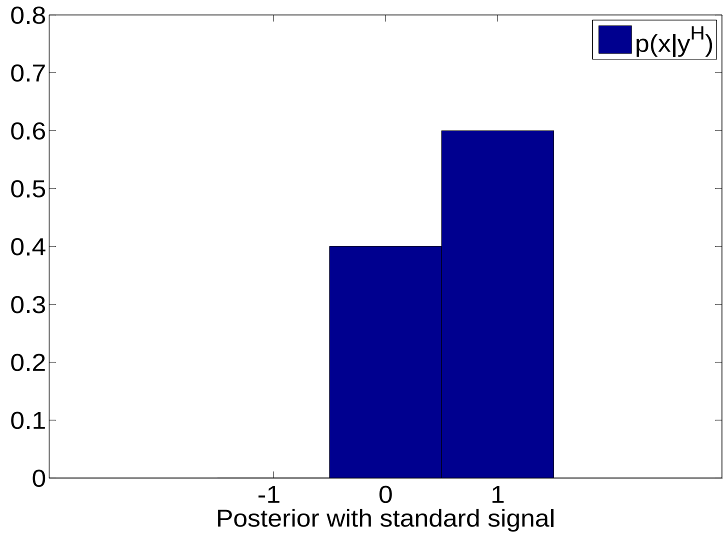

FIGURE 5. Distribution of x conditional on $y = y^H$ when y is always available

The same computations can be done for all other combinations of signals y and states x using the expression

$$p\left(x^i \mid y^j\right) = \frac{p\left(y^j \mid x^i\right) p\left(x^i \mid S=1\right)}{p\left(y^j\right)} \tag{1.41}$$

for $i, j \in \{H, M, L\}$.

2. A simple two-variable news selection model

This section will demonstrate how a man-bites-dog information structure can be generated from a set up with two latent variables in which a "newspaper" reports a signal about the variable that has had the most unusual realizations.

2.1. **Set up.** The basic set up is the following. There are two latent variables of interest; $x_1 \sim N(0, \sigma_{x1}^2)$ and $x_2 \sim N(0, \sigma_{x2}^2)$. There exists what we may think of as a newspaper that observes noisy signals z_1 and z_2 of x_1 and x_2

$$z_1 = x_1 + \varepsilon_1 : \varepsilon_1 \sim N(0, \sigma_{\varepsilon 1}^2) \tag{2.1}$$

$$z_2 = x_2 + \varepsilon_2 : \varepsilon_2 \sim N(0, \sigma_{\varepsilon 1}^2) \tag{2.2}$$

The newspaper will report either a noisy signal of x_1 or x_2 but due to page constraints is assumed to be unable to report both. The newspaper bases its decision on the man-bites-dog maxim and will thus report a signal about the variable that it thinks have had its most unusual realization. The newspaper does not observe the signal directly but forms an estimate of x_1 and x_2 based on the signals z_1 and z_2. Since

$$E\left[x_i \mid z_i\right] = \frac{\sigma_{xi}^2}{\sigma_{xi}^2 + \sigma_{\varepsilon i}^2} z_i : i \in \{1, 2\} \tag{2.3}$$

it will report the signal y_1

$$y_1 = x_1 + \eta_1 : \eta_1 \sim N(0, \sigma_{\eta 1}^2) \tag{2.4}$$

if $|z_1| > |z_2|$ and the signal y_2

$$y_2 = x_2 + \eta_2 : \eta_2 \sim N(0, \sigma_{\eta 2}^2) \tag{2.5}$$

otherwise. Define the indicator variable S to now take the value 1 when the signal y_1 is

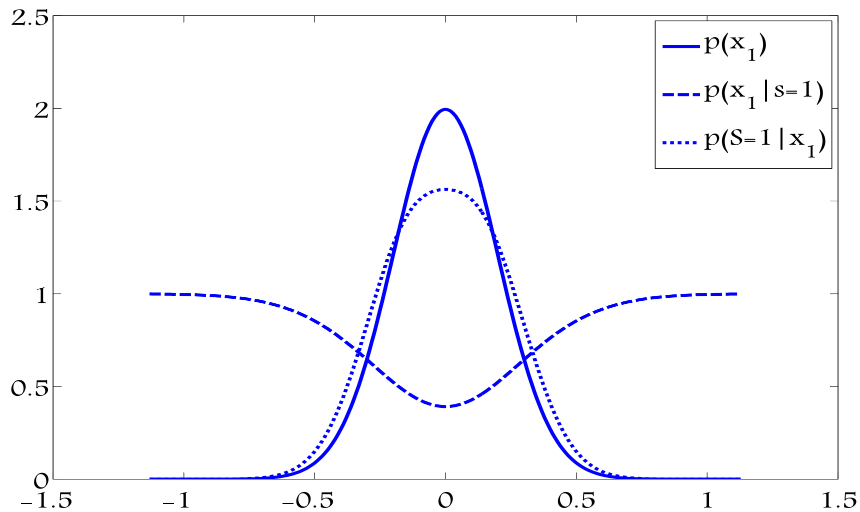

FIGURE 6. Conditional and unconditional distributions of x_1 and conditional probability of observing the signal y_1. (Note: Graph generated by scaling the standard deviation of all shocks by 0.2.)

available and zero otherwise.

2.2. **The implied conditional distributions.** Let the variable S take the value 1 when y_1 is reported and zero otherwise. We can use Bayes' rule

$$p(x_1 \mid S = 1) = \frac{p(S = 1 \mid x_1)\, p(x_1)}{p(S = 1)} \tag{2.6}$$

to find the distribution of x_1 conditional on the availability of the signal y_1. Since the information structure is symmetric in the two variables, we have that $p(S = 1) = 0.5$. By assumption, $p(x_1)$ is the standard normal distribution. To find $p(S = 1 \mid x)$ note that the probability of observing a signals about x_1 is the same as the probability that $|z_1| > |z_2|$ so that

$$\begin{aligned} p(S = 1 \mid x_1) &= p(|z_1| > |z_2| \mid x_1) && (2.7) \\ &= \int_{-\infty}^{\infty} p(\varepsilon_1)\left[1 - 2\left(1 - \Phi_{z2}\left(|x_1 + \varepsilon_1|\right)\right)\right] d\varepsilon_1 && (2.8) \end{aligned}$$

The expression (2.8) consists of the following parts. Φ_{z2} is the cdf of z_2 which is zero mean normally distributed with variance $\sigma^2_{x2} + \sigma^2_{\varepsilon 2}$. The quantity $1 - \Phi_{z2}(|x_1 + \varepsilon_1|)$ is thus the probability that $z_2 > |z_1|$, the quantity $2(1 - \Phi_{z2}(|x_1 + \varepsilon_1|))$ is the probability that $|z_2| > |z_1|$ so that $1 - 2(1 - \Phi_{z2}(|x_1 + \varepsilon_1|))$ equals the probability that $|z_1| > |z_2|$ for given x_1 and ε_1. Since we only condition on x_1 we need to integrate over the support of ε_1 weighting each probability $[1 - 2(1 - \Phi(|x_1 + \varepsilon_1|))]$ by the pdf of ε_1. The resulting integral (2.8) can be plugged into (2.6) to get the distribution of x_1 conditional on the signal y_1 being available. The three distributions that make up the expression (2.7) are plotted together in Figure 2.

We can see in the figure that the information structure has the man-bites-dog property that the conditional probability of observing a signal is increasing as realizations of x_1 further out in the tails are considered. Also, the distribution of x_1 conditional on $S = 1$ has fatter tails than the unconditional (normal) distribution of x_1.

Bayes' Rule can be used to find the distribution of x_1 conditional on a particular realization of y_1

$$p(x_1 \mid y_1) = \frac{p(y_1 \mid x_1)p(x_1 \mid S = 1)}{p(y_1)} \tag{2.9}$$

We can plug in the expressions

$$p(y_1 \mid x_1) = \frac{1}{\sigma_{\eta 1}\sqrt{2\pi}} e^{(y_1 - x_1)^2} \tag{2.10}$$

and

$$p(y_1) == \frac{1}{\sqrt{\sigma_{x1}^2 + \sigma_{\eta 1}^2}\sqrt{2\pi}} e^{\frac{y_1^2}{(\sigma_{x1}^2 + \sigma_{\eta 1}^2)}} \tag{2.11}$$

and $p(x_1 \mid S = 1)$ from (2.8) into (2.9) to get

$$p(x_1 \mid y_1) = \frac{\sqrt{\sigma_{x1}^2 + \sigma_{\eta 1}^2} e^{(y_1 - x_1)^2} \times \int_{-\infty}^{\infty} p(\varepsilon_1) \left[1 - 2\left(1 - \Phi_{z2}\left(|x_1 + \varepsilon_1|\right)\right)\right] d\varepsilon_1}{\sigma_{\eta 1} e^{\frac{y_1^2}{(\sigma_{x1}^2 + \sigma_{\eta 1}^2)}}} \tag{2.12}$$

This expression can be evaluated numerically for any particular realization of the signal y_1 and Figure 7 displays the distribution $p(x_1 \mid y_1 = -0.3)$.

Figure 7 illustrates an interesting property of the posterior beliefs conditional on a man-bites-dog signal: The posterior uncertainty about x_1 is larger than the prior uncertainty. This can be seen form the fact that the red dashed distribution is wider and less peaked than the solid blue unconditional (prior) distribution. We can contrast this result with what the conditional distribution would be if y_1 was always available. The posterior for that case is illustrated in Figure 8 by the dashed-dotted green distribution. There, the posterior distribution after observing a standard signal with the same precision as the man-bites-dog signal displays the standard property that it is more concentrated than the prior. We can also see that relative to the posterior distribution conditional on the man-bites-dog signal, the mean of the posterior is closer to the prior mean. That is, the conditional expectation of x_1 respond stronger to a man-bites-dog signal than to a standard signal with the same realized value and precision.

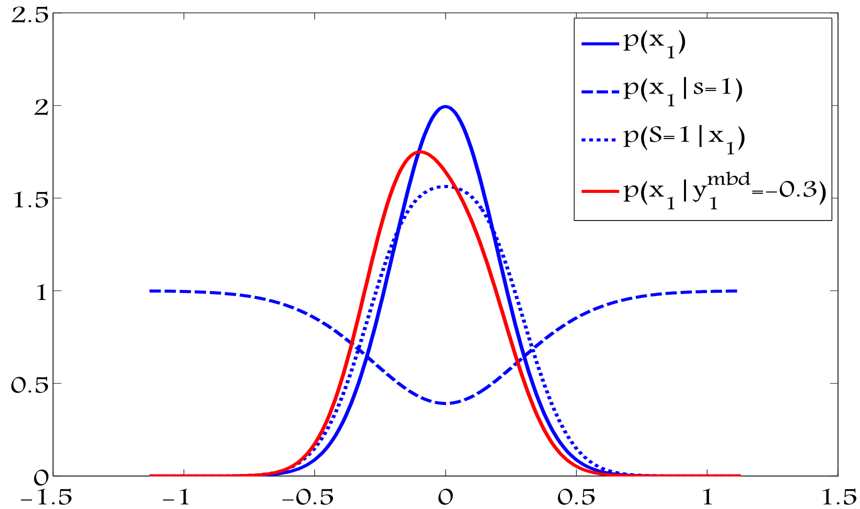

FIGURE 7. Conditional and unconditional distributions of x_1 and conditional probability of observing the signal y_1. (Note: Graph generated by scaling the standard deviation of all shocks by 0.2.)

2.3. **Comparison to alternative approaches.** Conceptually, the information structure analyzed in this section differs from the ex ante perspective taken by most of the existing literature on rational inattention, e.g. Sims (1998, 2003) and Mackowiak and Wiederholt (2009). In that literature, agents pay more attention to those variables that are most useful *on average*. In practise, this means that agents observe a noisy measure of the same linear combination of state variables in each period.

In contrast, here realizations of shocks matter for which signal is available. While we have not shown that this is optimal from the agents' perspective, it seems to capture one aspect of real world news reporting, namely that the attention of the news media shifts over time. Having said this, it is perhaps worth pointing out that there is nothing inherent in the rational inattention literature that makes an ex ante perspective necessary. For instance, Matejka (2011) develops a model of rational inattention in which it is optimal for agents to let the precision of signals depend on the realization of shocks. The information structure in that paper thus also depend on the realizations of shocks, but deterministically so. The

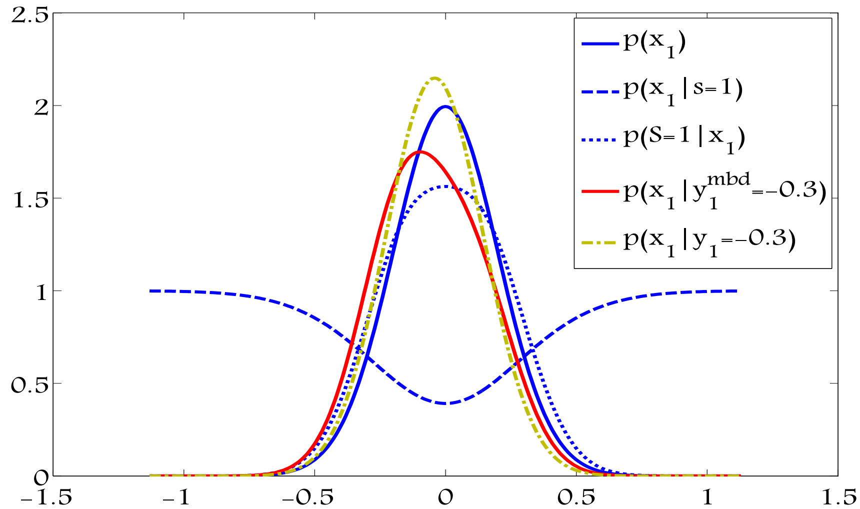

Figure 8. Conditional and unconditional distributions of x_1 and conditional probability of observing the signal y_1. (Note: Graph generated by scaling the standard deviation of all shocks by 0.2.)

availability of signals in Matejka's model is constant and thus do not carry any additional information about the distribution of the variables of interest.

3. Reverse engineering a tractable man-bites-dog information structure

In the previous section we constructed a man-bites-dog information structure where the conditional probabilities had to be computed numerically. In this section we show how Bayes' rule can be used to "reverse engineer" a tractable set up with analytical expressions for agents posterior beliefs. The approach we will take is the following. Since the agents know whether the signal y is available or not, they will never need to evaluate the unconditional distribution $p(x)$. Instead, agents will update from their conditional "prior" distribution $p(x \mid S)$ when observing a signal. We can then ensure tractability by directly specifying that the conditional distributions $p\left(x \mid S=0\right)$ and $p\left(x \mid S=1\right)$ are Gaussian. By applying the results of Proposition 1 "in reverse" when we parameterize these conditional distributions we can ensure both tractability and that the inequalities in Definition 1 are satisfied.

Throughout this section, agents indexed by $j \in (0,1)$ want to form an estimate of the latent variable x conditional on all available information. There are two types of signals. In every period, agent j observes a private signal x_j which is the sum of the true x plus an idiosyncratic noise term

$$x_j = x + \varepsilon_j : \varepsilon_j \sim N\left(0, \sigma_\varepsilon^2\right) \;\; \forall \;\; j. \tag{3.1}$$

where the variance of the idiosyncratic noise term ε_j is common across agents. When available, the man-bites-dog signal y

$$y = x + \eta : \eta \sim N\left(0, \sigma_\eta^2\right). \tag{3.2}$$

is observed by all agents. The fact that all agents observe y when it is available is common knowledge (though this does not really matter until later).

Now, specify x conditional on S to be normally distributed

$$p\left(x \mid S = 0\right) = N\left(0, \sigma^2\right) \tag{3.3}$$

$$p\left(x \mid S = 1\right) = N\left(0, \gamma\sigma^2\right) \tag{3.4}$$

so that the unconditional distribution $p(x)$ is a mixture normal

$$x \sim (1-\omega)\, N\left(0, \sigma^2\right) + \omega N\left(0, \gamma\sigma^2\right) \tag{3.5}$$

The parameter ω then determines how often the signal y is observed in the unconditional sense, i.e. $\omega = p(S = 1)$.

3.1. **Verifying Definition 1.** The probability density $p(x)$ of a mixture of two normals both centered at zero is decreasing in the absolute value of x. Larger realizations of x are thus more unusual so that for y to be a man-bites-dog signal we need the probability $p\left(S = 1 \mid x\right)$ to be increasing in the absolute value of x. To find such a parameterization, start by dividing

Bayes' rule for conditional probabilities for $p(S = 1 \mid x)$

$$p(S = 1 \mid x) = \frac{p(x \mid S = 1)p(S = 1)}{p(x)} \tag{3.6}$$

and $p(S = 0 \mid x)$

$$p(S = 0 \mid x) = \frac{p(x \mid S = 0)p(S = 0)}{p(x)} \tag{3.7}$$

with each other to get

$$\frac{p(S = 1 \mid x)}{p(S = 0 \mid x)} = \frac{p(x \mid S = 1)}{p(x \mid S = 0)} \frac{p(S = 1)}{p(S = 0)} \tag{3.8}$$

Substitute in the distributional assumptions for $p(x \mid S = 1)$ and the unconditional probabilities of observing y

$$\frac{p(S = 1 \mid x)}{p(S = 0 \mid x)} = \frac{\frac{1}{\sqrt{\gamma}\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{x^2}{\gamma\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}} \frac{\omega}{1-\omega} \tag{3.9}$$

Rearrange and simplify to get

$$\frac{p(S = 1 \mid x)}{1 - p(S = 1 \mid x)} = \frac{\omega}{1-\omega} \frac{1}{\sqrt{\gamma}} e^{\left(1-\frac{1}{\gamma}\right)\frac{x^2}{2\sigma^2}} \tag{3.10}$$

where we also used that $p(S = 0 \mid x) = 1 - p(S = 1 \mid x)$. The term in brackets in the exponent on the right hand side of (3.10) that multiplies the square of x is positive if $\gamma > 1$. Imposing this restriction on γ thus ensures that the conditional probability $p\,(S = 1 \mid x)$ of observing the signal y is increasing in the absolute value of x. It is also clear form the expression that setting $\gamma = 1$ makes the probability of observing γ independent of x.

3.1.1. *Manipulating the probability of observing y.* Choosing ω, γ and σ^2 let us manipulate the shape of the conditional probability of observing the signal y. For instance, observing y can be made a rare event by setting ω close to zero. The parameter γ determines how informative the availability of y is about the conditional distribution of x. With a low ω and a γ close to 1, even though it is very rare to observe y, doing so does not mean that unusual realizations are significantly more likely. This can be seen from that the ratio of the conditional densities

$$\frac{p(x \mid S=1)}{p(x \mid S=0)} = \frac{1}{\sqrt{\gamma}} e^{\left(1-\frac{1}{\gamma}\right)\frac{x^2}{2\sigma^2}} \tag{3.11}$$

is close to 1 when $\gamma \approx 1$. That is, with the parameter γ approximately equal to unity, observing y is does not imply a conditional distribution of x that is very different from its unconditional distribution. The parameter γ also determines the relative probability of a realization of x near the zero mean. When $\gamma = 0$, the ratio (3.11) is simply given by $1/\sqrt{\gamma}$ so that realizations near the (zero) mean are relatively much less likely when γ is large.

While manipulating the parameters ω and γ we may want to treat the unconditional variance of x as a primitive that we want to keep fixed. For all values of ω and γ this is always possible since the variance of the mixture normal distribution (3.5) is given by

$$\sigma_x^2 = \omega\gamma\sigma^2 + (1-\omega)\,\sigma^2 \tag{3.12}$$

and thus provides sufficient flexibility to hold σ_x^2 fixed by scaling σ^2.

3.2. **Posterior beliefs.** Given the definitions (3.1) and (3.1) of the signals x_j and y and the distributional assumptions (3.3) - (3.4), agent $j's$ conditional expectation of x are given by standard formulas for multiple signals with independent Gaussian noise processes. Denoting agent j's information set Ω_j^0 when $S=0$ and Ω_j^1 when $S=1$ we have

$$E\left(x \mid \Omega_j^0\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}} x_j \tag{3.13}$$

and

$$E\left(x \mid \Omega_j^1\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} y \tag{3.14}$$

The weights on the signals are determined by the relative precision of the individual signals and the respective conditional distribution. The posterior variances are also standard and given by

$$E\left[x - E\left(x \mid \Omega_j^0\right)\right]^2 = \left(\sigma_\varepsilon^{-2}+\sigma^{-2}\right)^{-1} \tag{3.15}$$

or

$$E\left[x - E\left(x \mid \Omega_j^1\right)\right]^2 = \left(\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}\right)^{-1}. \tag{3.16}$$

depending on wether the signal y is available or not.

3.3. **Properties of conditional expectations with *man-bites-dog* signals.** The set up described above allows us to prove two results that may initially appear counterintuitive, but are natural once the implications for agent's conditional distributions of a man-bites-dog information structure is understood. First, the posterior uncertainty can be larger after the signal y has been observed, compared to when it has not been observed. Secondly, the dispersion of expectations about x may increase after y is observed, even though y is a public signal.

Proposition 2. *The posterior uncertainty about x can be larger when the signal y is observed relative to when it is not. I.e. there are parameter values such that the inequality*

$$E\left[x - E\left(x \mid \Omega_j^0\right)\right]^2 < E\left[x - E\left(x \mid \Omega_j^1\right)\right]^2 \tag{3.17}$$

holds.

Proof. Directly comparing the posterior variances (3.15) and (3.16) implies that the proposition holds if

$$\sigma_\varepsilon^{-2} + \sigma^{-2} > \sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2} \tag{3.18}$$

Rearranging this expression gives

$$\sigma_\eta^2 > \frac{\sigma^2}{(1-\gamma^{-1})} \tag{3.19}$$

so that if $\gamma > 1$ there exists some $\sigma_\eta^2 > \sigma^2$ such that the inequality in the proposition holds. □

Proposition 2 states that if the signal y is noisy enough and $\gamma > 1$, the posterior variance may be larger when y is observed compared to when it is not. This result is an implication of the inequality (1.5) that states that more unusual realizations of x are relatively more likely when a man-bites-dog signal is available. This effect is illustrated in Figure 9, where it can be seen that the distribution of x has more mass in the tails conditional on y being available. If the variance of the noise σ_η^2 in y is large enough, this effect will dominate any increase in precision in agents beliefs from observing the realized value of y.

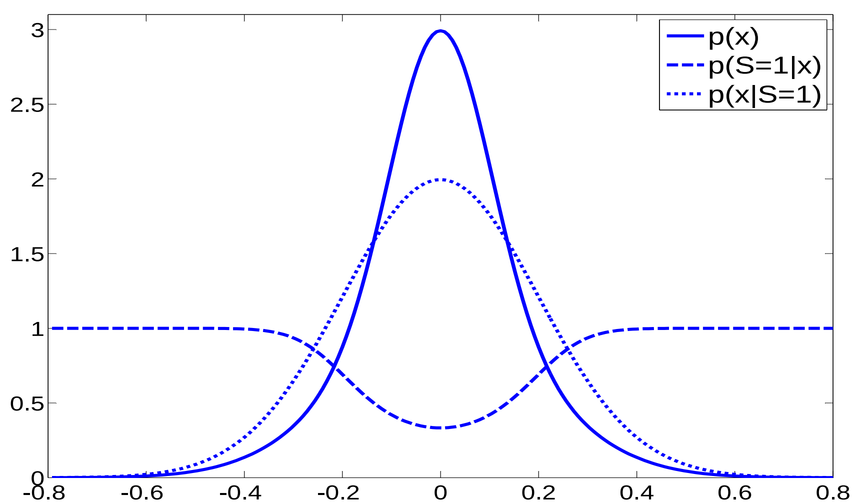

FIGURE 9. Unconditional distribution of x, conditional probability of observing the signal y and the implied conditional distribution of x.

For a fixed variance of the noise in the signal y, the inequality in the proposition is more likely to hold if γ is large. A large γ implies that only very unusual realizations of x are significantly more likely to generate the signal y. The distribution $p(x \mid S = 1)$ then has lot more probability mass in the tails relative to the unconditional distribution $p(x)$. For a large enough γ, the signal y then do not have to be very noisy for this effect to dominate. However, since the right hand side of (3.19) has a minimum of σ^2 when $\gamma \to \infty$, that $\sigma_\eta^2 > \sigma^2$ is a necessary condition for the inequality in the proposition to hold.

Since uncertainty may increase after observing y one may think that risk averse agents would be better off if the signal y was never available. This is not the case. It can be shown that even when the private signal x_j is uninformative, the unconditional expectation of the posterior variance of agents' beliefs is strictly smaller than the unconditional variance σ_x^2 as long as the man-bites-dog signal is not infinitely noisy. This is related to a result from information theory stating that in general, it is possible that some realizations of signals may increase entropy, though on average entropy must decrease when conditioning on more information (see Theorem 2.6.5 of Cover and Thomas 2006).[2]

Corollary 1. *When the inequality*

$$\sigma_\eta^2 > \frac{\sigma^2}{(1-\gamma^{-1})} \tag{3.20}$$

holds, the cross sectional dispersion of expectations about x is larger when y is observed compared to when it is not.

The proof follows directly from that the denominator in the weight on the private signal is the same as the denominator in the posterior variances. The cross sectional dispersion is increasing in the weight on the private signal, holding the variance of the idiosyncratic noise constant. The same conditions that deliver a higher posterior variance thus also deliver more weight on the private signal and the intuition is also similar. If the public noise variance is high and the conditional probability of a tail event is high, agents will put more weight on other (e.g. private) sources of information.

It is straight forward to show that the total weight agents put on all signals increases when $S = 1$.

Proposition 3. *The (cross-sectional) average expectation of x responds stronger to x when $S = 1$ than when $S = 0$.*

[2] I am indebted to Mirko Wiederholt for pointing out this link to me.

Proof. We need to show that the sum of the coefficients on the private signal x_j and the public signal y in the conditional expectation

$$E\left(x \mid \Omega_j^1\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} y \tag{3.21}$$

when $S = 1$ is larger than the coefficient on the private signal

$$E\left(x \mid \Omega_j^0\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}} x_j \tag{3.22}$$

when $S = 0$. Simply comparing the expected average expectation conditional on x for $S = 0$

$$E\left(\int E\left[x \mid \Omega_j^0\right] dj \mid x\right) = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}} x_j dj \tag{3.23}$$

$$= \left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) x \tag{3.24}$$

and $S = 1$

$$E\left(\int E\left(x \mid \Omega_j^1\right) dj \mid x\right) = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x_j dj \tag{3.25}$$

$$+ \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x \tag{3.26}$$

$$= \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) x \tag{3.27}$$

means that the proposition is true if the inequality

$$\left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) < \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) \tag{3.28}$$

holds. The last expression can with a little algebra be rearranged to

$$\gamma^{-1} < 1 + \sigma_\eta^{-2} \tag{3.29}$$

which is always true since $\gamma > 1$ and $\sigma_\eta^{-2} > 0$. $\square$

4. Higher order expectations: Concepts and notation

Above, the implications of man-bites-dog signals for agents' posterior beliefs were analyzed. in order to understand how such information structures affect economic decisions, we will embed the information structure from the previous section in the strategic game of Morris and Shin (2002). Before we do that though, it is helpful to first introduce a notation for higher order expectations.

We can denote agent j's first order expectation of x_t at time t

$$x^{(1)}(j) \equiv E\left[x \mid \Omega_j\right] \tag{4.1}$$

The average first order expectation is obtained by taking averages of (4.1) across agents

$$x^{(1)} \equiv \int E\left[x \mid \Omega_j\right] \quad dj \tag{4.2}$$

The average second order expectation is obtained by taking the average of agents' expectations of (4.2)

$$x^{(2)} \equiv \int E\left[x^{(1)} \mid \Omega_j\right] \; dj \tag{4.3}$$

The average contemporaneous second order expectation of x_t thus is the average expectation at time t of the average expectation at time t of the value of x_t. We can generalize this notation to the k^{th} order expectation of x_t

$$x^{(k)} \equiv \int E\left[x^{(k-1)} \mid \Omega_j\right] \quad dj \tag{4.4}$$

Define the zero order expectation of θ_t as the actual value of the variable

$$x^{(0)} \equiv x \tag{4.5}$$

In general

$$x^{(k)} \neq x^{(k+l)} \tag{4.6}$$

for $l \neq 0$.

5. Man-bites-dog signals in a beauty contest model

Above, the implications of a man-bites-dog information structure for agents' beliefs about x were analyzed in some detail, but there were no economic decisions made by the agents. To remedy this shortcoming, we now introduce the information structure from Section 4 above into the beauty contest model of Morris and Shin (2002). This simple model will help us build intuition for the how a man-bites-dog information structure affects economic decisions in the presence of strategic complementarities.

5.1. **A beauty contest model.**

The model of Morris and Shin (2002) consists of a utility function U_j for agent j

$$U_j = -(1-r)(a_j - x)^2 - r\left(L_j - \overline{L}\right) \tag{5.1}$$

where a_j is the action taken by agent j. The first quadratic term in the utility function implies that agent j wants to take an action that is close to the value of the unobserved variable x. The second term introduces a strategic motive since

$$L_j \equiv \int (a_i - a_j)^2 \, di \tag{5.2}$$

and

$$\overline{L} \equiv \int L_i di \tag{5.3}$$

Maximizing the expected value of the utility function (5.1) results in a first order condition for agent j given by

$$a_j = (1-r) E[x \mid \Omega_j] + rE[\overline{a} \mid \Omega_j] \tag{5.4}$$

where $\overline{a}$ is the cross-sectional average action

$$\overline{a} \equiv \int a_i di \tag{5.5}$$

For a positive value of r, agent j thus want to take an action that is close to the true value of x as well as close to the average action taken by other agents. The relative weight of these two objectives is determined by the parameter r. This basic structure is identical to that of the model in Morris and Shin (2002). As shown by Angeletos, Iovino and La'O (2011), agent $j's$ first order condition (5.4) is isomorphic to that of a firm in a simple business cycle model with monopolistic competition and dispersed information. In that setting, the action a_j corresponds to the optimal level of firm $j's$ output and the parameter r is a composite function of the parameters governing the curvature of the utility function and the elasticity of substitution between differentiated goods.

Instead of using the method of undetermined coefficients employed by Morris and Shin in the original paper, we will use a method that explicitly phrases the the average action $\overline{a}$ as a function of higher order expectations of x.

In rational expectations models, (first order) expectations are pinned down by the structure of the model. That is, an agent's expectations should be the mathematical expectation of the variable in question, conditional on the information set available to the agent. The underlying assumption we make is thus that agents know the structure of the economy, that is, agents know the functional form and true parameter values of the model. Similarly, second order knowledge of rationality can be used to pin down second order expectations. That is, a rational agent's expectations can also be predicted, and treated as a random variable like any other. If an agent wants to form an expectation about another agents expectation, and knows that the other agent is rational, then second order expectation will be the rational expectation conditional on the expected information set of the other agent. A similar logic can be applied to third and higher order expectations. We now show how the structure imposed by common knowledge of rational expectations can be used to solve Morris and Shin's model.

Start by taking averages of (5.4) to get

$$\overline{a} = (1-r)\, x^{(1)} + r\overline{a}^{(1)} \tag{5.6}$$

where $\overline{a}^{(1)}$ is the average (first order) expectation of the average action $\overline{a}$, i.e.

$$\overline{a}^{(1)} \equiv \int E\left[\overline{a} \mid \Omega_j\right] dj \tag{5.7}$$

Substituting the terms in (5.6) into (5.7) we get

$$\begin{aligned}
\overline{a}^{(1)} &= \int E\left[(1-r)\, x^{(1)} + r\overline{a}^{(1)} \mid \Omega_j\right] dj && (5.8)\\
&= (1-r)\, x^{(2)} + r\overline{a}^{(2)} && (5.9)
\end{aligned}$$

By taking the average expectation of (5.8) and so on, gives a general expression for the k order expectation of the average action

$$\overline{a}^{(k)} = (1-r)\, x^{(k+1)} + r\overline{a}^{(k+1)} \tag{5.10}$$

Now, repeated substitution of (5.10) into (5.6) and simplifying gives the geometric sum

$$\overline{a} = (1-r) \sum_{k=1}^{\infty} r^{k-1} x^{(k)} \tag{5.11}$$

The expression (5.11) describes the optimal average action regardless of whether the signal y is available or not.

5.2. **Higher order expectations when $S = 0$.** When $S = 0$ we have that

$$E\left(x \mid \Omega_j^0\right) = g_0 x_j \tag{5.12}$$

where

$$g_0 \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}$$

Taking averages across agent we get

$$x^{(1)} = \int E\left(x \mid \Omega_j^0\right) dj \tag{5.13}$$

$$= g_0 x + g_0 \int \varepsilon_j dj \tag{5.14}$$

$$= g_0 x \tag{5.15}$$

since $\int \varepsilon_j dj = 0$. Taking expectations of the average first order expectation of x gives

$$E\left(x^{(1)} \mid \Omega_j^0\right) = E\left(g_0 x \mid \Omega_j^0\right) \tag{5.16}$$

$$= g_0 E\left(x \mid \Omega_j^0\right) \tag{5.17}$$

$$= g_0^2 x_j \tag{5.18}$$

Again, averaging across agents we get an expression for the average second order expectation about x

$$x^{(2)} = g_0^2 x \tag{5.19}$$

repeating the same steps to get the average expectation about $x^{(2)}$ and so on yields a general expression for the k order expectation when $S = 0$

$$x^{(k)} = g_0^k x \tag{5.20}$$

5.3. **Higher order expectations when** $S = 1$**.** When agents observe both x_j and y the conditional expectation is given by

$$E\left(x \mid \Omega_j^1\right) = E\left(x \mid y\right) + E\left(x \mid \left[x_j - E\left(x \mid y\right)\right]\right) \tag{5.21}$$

$$= g_y y + g_x \left(x_j - g_y y\right) \tag{5.22}$$

where

$$g_y \equiv \frac{\sigma_\eta^{-2}}{\sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}, \quad g_x \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} \tag{5.23}$$

Taking averages

$$x^{(1)} = \int E\left(x \mid \Omega_j^1\right) dj \tag{5.24}$$

$$= g_y y + g_x \left(x - g_y y\right) \tag{5.25}$$

Agent j's expectation of the average first order expectation is then given by

$$E\left(x^{(1)} \mid \Omega_j^1\right) = E\left(g_y y + g_x \left(x - g_y y\right) \mid \Omega_j^1\right) \tag{5.26}$$

$$= g_y y + g_x \left(g_y y + g_x \left(x_j - g_y y\right) - g_y y\right) \tag{5.27}$$

$$= g_y y + g_x \left(g_x \left(x_j - g_y y\right)\right) \tag{5.28}$$

$$= g_y y + g_x^2 \left(x_j - g_y y\right) \tag{5.29}$$

$$x^{(2)} = g_y y + g_x^2 \left(x - g_y y\right) \tag{5.30}$$

repeating the procedure gives the general expression for a k order expectation

$$x^{(k)} = g_y y + g_x^k \left(x - g_y y\right) \tag{5.31}$$

By the expression for the k order expectation (5.31) we can see that

$$\lim_{k \to \infty} x^{(k)} = g_y y \tag{5.32}$$

since $0 < g_x < 1$. Thus, just as in Morris and Shin's model, higher order expectations tend to be dominated by the public signal y (when available) as the order of expectation increases.

5.4. **The average action as a function of x and y.** In order to determine how the average action $\overline{a}$ is affected by the man-bites-dog structure, substitute the expressions for the higher order expectations (5.20) and (5.31) into the average action expression (5.11). We then have that

$$\overline{a} = (1-r)\sum_{k=1}^{\infty} r^{k-1} g_0^k x \tag{5.33}$$

$$= \frac{(1-r)\,g_0}{1-rg_0} x \tag{5.34}$$

when $S=0$ and

$$\overline{a} = (1-r)\sum_{k=1}^{\infty} r^{k-1}\left(g_y y + g_x^k\left(x - g_y y\right)\right) \tag{5.35}$$

$$= \frac{(1-r)\,g_x}{1-rg_x}\left(x - g_y y\right) + g_y y \tag{5.36}$$

$$= \frac{(1-r)\,g_x}{1-rg_x} x + \left(1 - \frac{(1-r)\,g_x}{1-rg_x}\right) g_y y \tag{5.37}$$

when $S=1$.We can now prove the following.

Proposition 4. *The response of the average action $\overline{a}$ to a given value of x is stronger when the signal y is available.*

Proof. We need to prove that

$$\frac{(1-r)\,g_0}{1-rg_0} < \frac{(1-r)\,g_x}{1-rg_x} + \left(1 - \frac{(1-r)\,g_x}{1-rg_x}\right) g_y \tag{5.38}$$

Divide everywhere by $(1-r)$ to get

$$\frac{g_0}{1-rg_0} < \frac{g_x}{1-rg_x} + \frac{g_y}{1-r} - \frac{g_x g_y}{1-rg_x} \tag{5.39}$$

and rearrange to get

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y}{1-rg_x} + \frac{g_y}{1-r} \tag{5.40}$$

Now since $0 < g_x < 1$

$$\frac{g_y}{1-rg_x} < \frac{g_y}{1-r} \tag{5.41}$$

It is thus sufficient to prove that

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y + g_y}{1-rg_x} \tag{5.42}$$

Now if $g_x > g_0$ we have that

$$\frac{(1-r)\,g_0}{1-rg_0} < \frac{(1-r)\,g_x}{1-rg_x} \tag{5.43}$$

which would be sufficient since the second term on the right hand side of (5.38) is positive. If $g_x < g_0$ it is sufficient to prove that

$$g_0 < g_x - g_x g_y + g_y \tag{5.44}$$

By equating the two expressions (3.14) and (5.31) for the first order expectation of x

$$\frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}x + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}y = g_x - g_x g_y + g_y \tag{5.45}$$

and using that

$$g_0 = \left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}}\right) \tag{5.46}$$

and

$$g_x - g_x g_y + g_y = \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}\right) \tag{5.47}$$

we arrive at the same inequality

$$\left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}}\right) < \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}\right) \tag{5.48}$$

that was demonstrated to hold generally in Proposition 3. □

The proposition holds for all parameter values, including when $\gamma = 1$ and is thus true partly because expectations (of all orders) simply respond stronger when there are more signals available. However, the effect is reinforced by the man-bites-dog information structure.

We can also use the expression (5.37) to better understand how the impact of noise shocks in the public signal depends on the man-bites-dog information structure. When the signal

y is available, the impact of a noise shock is given by the coefficient on y in (5.37) since $y = x + \eta$. The maximum impact of a noise shock is achieved for $r = 1$ and large values of γ. In the limit, as $\gamma \to \infty$ the coefficient $g_y \to 1$ and the average action responds 1-for-1 to a noise shock. This limit can be achieved for any finite man-bites-dog noise variance σ_η^2. Since we can increase γ while holding the unconditional variance of x fixed by decreasing ω we can have an arbitrarily large variance of $\overline{a}$ caused by noise, conditional on y being available. Stated differently, if only very unusual realizations of x trigger the availability of a man-bites-dog signal, the response to noise in that signal can be very large. Of course, since we have to make the event that y is observable very rare in order to hold the variance of x fixed, there is still a bound on how much of the unconditional variance of $\overline{a}$ that can be caused by the noise in the public signal.

Lastly, while it is true that strategic complementarities tend to dampen the impact of x on $\overline{a}$, they can magnify the difference between the response of $\overline{a}$ when the signal y is available compared to when it is not. General results can only be determined numerically, but a limit example illustrates the point. When agents care only about taking an action that is close the average action, i.e. when $r = 1$, the average action does not respond at all in absence of the public signal y. When the signal y is available, the expression for the average action (5.37) simplifies to $g_y y$ and the ratio of impact on $\overline{a}$ of x in the two cases is infinite (or zero).

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