ECONOMETRIC METHODS II: TIME SERIES

EXERCISE QUESTIONS

These questions are intended to be representative of the type of question that can be asked in the midterm.

Cointegration

(1) Define stationarity.
(2) Define cointegrated processes.
(3) For the system

$$\begin{aligned} y_{1t} &= \gamma y_{2t} + u_{1t} \\ y_{2t} &= y_{2t-1} + u_{2t} \end{aligned}$$

show that the y_1 and y_2 are cointegrated.
   (a) What is the cointegrating vector?
   (b) How many cointegrating vectors are there?
   (c) How many linearly independent cointegrating vectors are there?
(4) The variables in y_t are cointegrated and can be represented as VAR(2) in levels

$$\mathbf{y}_t = \alpha + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \varepsilon_t$$

   (a) Find the Vector Error Correction (VECM) representation of y_t
   (b) Explain intuitively why we cannot just difference the data.
   (c) How can the coefficients in the VECM be estimated?
   (d) How should the coefficients in the VECM be estimated if the cointegrating vector is known? Why?
(5) Show with a simple example why the estimates of the cointegrating relationship are unbiased, even in the presence of contemporaneous correlation between the shock and the regressors.
(6) Describe how to test whether a collection of time series are cointegrated.

Structural VARS

(1) Why can't we estimate A_0 and A_1 in the structural form

$$A_0 \mathbf{y}_t = A_1 \mathbf{y}_{t-1} + \mathbf{u}_t$$

directly?
(2) Describe how to identify A_0 and A_1 using contemporaneous restrictions.
   (a) How many restrictions do you need if y_t is n dimensional?
(3) Describe how to identify A_0 and A_1 using long run restrictions.
   (a) How many restrictions do you need if y_t is n dimensional?

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(4) Derive an operational (i.e. a useful) expression for the variance of y_t through the MA representation
(5) Derive an operational (i.e. a useful) expression for the variance of y_t using that u_t is orthogonal to y_{t-1}.
(6) Find the impulse response function for the structural form above.
(7) Describe a procedure to decompose the variance of y_t into the fractions caused by the individual shocks in the vector u_t
(8) Structural VARS: What's good about them?
(9) Structural VARS: What's not so good about them?

Principal components and factor models

(1) Describe a scree plot and how they can be used to determine the number of factors.
(2) Let Y_t be the $N \times T$ dimensional data set with covariance $E(Y_t Y_t') = W\Omega W'$ where $W\Omega W'$ is the eigenvector/value decomposition. How can the principal component factors be extracted from the data?
(3) Let

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t \\ Z_t &= DX_t + \mathbf{v}_t \end{aligned}$$

describe a state space system where $E(Y_t Y_t') \approx DE(X_t X_t') D'$. Describe how the state space can be "rotated" into the principal component representation

$$\begin{aligned} F_t &= \Phi F_{t-1} + e_t : e_t \sim N(0, \Sigma_e) \\ Z_t &= BF_t \end{aligned}$$

i.e. find Φ, Σ_e and B.

State Space Models and the Kalman Filter

(1) Find the state space form

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t \\ Z_t &= DX_t + \mathbf{v}_t \end{aligned}$$

of the VARMA(p,q)

$$\begin{aligned} y_t &= \Phi_1 y_{t-1} + ...\Phi_p y_{t-p} \\ &\quad + \varepsilon_t + \Psi_1 \varepsilon_{t-1} + ... + \Psi_q \varepsilon_{t-q} \end{aligned}$$

where $Z_t = y_t$.

(2) For the scalar process

$$\begin{aligned} x_t &= \rho x_t + u_t \\ z_t &= x_t + v_t \\ \begin{bmatrix} u_t \\ v_t \end{bmatrix} &\sim N\left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}\right) \\ x_{0|0} &= \overline{x}_0 \\ E\left(\overline{x}_0 - x_0\right)^2 &= p_{0|0} \end{aligned}$$

find the Kalman gain k_t such that $x_{t|t}$ given by

$$x_{t|t} = \rho x_{t-1|t-1} + k_t\left[z_t - \rho x_{t-1|t-1}\right]$$

is the linear minimum variance estimate of x_t conditional on $\overline{x}_0$ and the history of z_t.

(3) What is k_t if $\sigma_v^2 = 0$? Interpret.
(4) What is k_t if $\sigma_v^2 = \infty$? Interpret.
(5) What is k_t if $\sigma_u^2 = 0$? Interpret.
(6) What is k_t if $\sigma_u^2 = \infty$? Interpret.
(7) Describe how k_t depends on the observations $z_t, z_{t-1}, ...z_1$
(8) What is the log likelihood function of the state space system

$$\begin{aligned} X_t &= A_t X_{t-1} + C_t \mathbf{u}_t \\ Z_t &= D_t X_t + \mathbf{v}_t \end{aligned}$$

?

(9) For the general state space system above, find the impulse response function of the elements of Z_t to the individual shocks in u_t, i.e. find

$$\frac{\partial z_{m,t+s}}{\partial u_{n,t}}$$

where

$$Z_t = \begin{bmatrix} z_{1t} \\ z_{2t} \\ \vdots \\ z_{Mt} \end{bmatrix}$$

and

$$u_t = \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{nt} \end{bmatrix}$$

Numerical Maximization

(1) Consider the model

$$y_t = \rho_1 y_{-1} + \rho_2 y_{t-2} + u_t$$

(a) What is the vector θ of parameters?
(b) Describe a grid search procedure to find the MLE $\widehat{\theta}$

(c) What are the advantages and disadvantages of grid search?
(d) Describe the steepest ascent method to find the MLE $\widehat{\theta}$
(e) What are the advantages and disadvantages of steepest ascent?
(f) Describe a simulated annealing algorithm that can be used to find the MLE $\widehat{\theta}$
(g) What are the advantages and disadvantages of simulated annealing?

Bayesian Methods

(1) What is the fundamental difference between frequentist and Bayesian methods?
(2) Describe the Random Walk Metropolis Algorithm
   (a) What is good choice of initial value θ_0 for the Markov chain of parameter vectors?
   (b) What is a good choice of covariance matrix for generating candidate draws θ^*?
   (c) Describe convergence of the Markov chain and how to check for it
(3) Taking a simulated posterior distribution as given, explain how to construct probability intervals of impulse response functions