

**TOPICS IN MACROECONOMICS: MODELLING INFORMATION,
LEARNING AND EXPECTATIONS**

EXERCISE QUESTIONS 2009

These questions are representative of the questions that will be in the midterm.

1. BASIC METHODS

- (1) Consider the two random vectors X and Y

$$X = \theta + u$$

$$y = \theta + x$$

with $E[\theta\theta'] = \sigma_\theta^2$, $E[uu'] = \sigma_u^2$, $E(u\theta') = 0$, $E[xx'] = \sigma_x^2$, $E[x\theta'] = 0$ and $E[xu'] = 0$. Find the minimum variance linear estimator of X conditional on Y using the properties of orthogonal projections.

- (2) Put the ARMA(1,2) process

$$y_t = A_1 y_{t-1} + C_0 \mathbf{u}_t + C_1 \mathbf{u}_{t-1} + C_2 \mathbf{u}_{t-2} \quad (1.1)$$

in the form

$$X_t = \mu_X + AX_{t-1} + C\mathbf{u}_t \quad (1.2)$$

- (3) Derive an operational procedure for finding the auto covariance $\Sigma_{xx} \equiv E[X_t - \mu_X][X_t - \mu_X]'$ for a system of the form (1.2). Be specific about additional assumptions made.
- (4) Let $\mathcal{P}(X|Z, Y)$ denote the orthogonal projection of X on the space spanned by Z and Y . Show that

$$\mathcal{P}(X|Z, Y) = \mathcal{P}(X|Z) + \mathcal{P}(X|Y) \quad (1.3)$$

if $\mathcal{P}(Y|Z) = 0$.

2. THE KALMAN FILTER

(1) For the state space system

$$X_t = AX_{t-1} + C\mathbf{u}_t \quad (2.1)$$

$$Z_t = DX_t + \mathbf{v}_t \quad (2.2)$$

$$\begin{bmatrix} \mathbf{u}_t \\ \mathbf{v}_t \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} CC' & \mathbf{0}_{n \times l} \\ \mathbf{0}_{l \times n} & \Sigma_{vv} \end{bmatrix}\right) \quad (2.3)$$

derive the first period Kalman gain K_1 in

$$X_{1|1} = AX_{0|0} + K_1(Z_1 - DAX_{0|0}) \quad (2.4)$$

taking $X_{0|0}$ and $P_{0|0} \equiv E(X_0 - X_{0|0})(X_0 - X_{0|0})'$ as given.

3. THE LUCAS AER (1973) ISLAND MODEL

(1) Interpret the slope parameter $\theta\gamma$ in the supply curve

$$y_t = \theta\gamma [P_t - \bar{P}_t] \quad (3.1)$$

in the Lucas Island model.

- (2) Explain the predictions made by the Lucas Island model regarding the correlation between cyclical output and inflation in different countries.
- (3) Explain why information imperfections lead to higher output volatility in the Lucas Island model

4. LORENZONI'S AER (2009) ISLAND MODEL

(1) Describe how Lorenzoni's model differ from the Lucas model

- (2) What is the modeling device Lorenzoni uses to avoid that a firm's own output reveals the aggregate shock perfectly?

5. PRIVATE AND PUBLIC INFORMATION

- (1) Consider the unobservable variable θ given by

$$\theta \sim N(0, \sigma_\theta^2) \quad (5.1)$$

Agents (indexed by j) observe a private noisy signal of θ given by

$$z(j) = \theta + \varepsilon(j) \quad (5.2)$$

$$\eta(j) \sim N(0, \sigma_\varepsilon^2) \forall j$$

That is, all agents receive an equally precise signal of θ but agent j only observes his own signal $z(j)$. Define

$$\theta^{(k)} \equiv \int E[\theta^{(k-1)} | z(j)] dj \quad (5.3)$$

$$\theta^{(0)} \equiv \theta \quad (5.4)$$

Find an expression for $\theta^{(k)}$. What is the limit as $k \rightarrow \infty$?

- (2) Redo 1) but with public signal

$$y = \theta + \eta$$

and θ non-stochastic (as in Morris and Shin 2002).

6. MORRIS AND SHIN AER (2002)

- (1) Utility of agent i is given by

$$U_i = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L})$$

where a_i is the action taken by agent i and

$$L_i = \int (a_j - a_i)^2 dj$$

and

$$\bar{L} = \int L_j dj$$

Agents observe two signals of θ . The public signal y

$$y = \theta + \eta$$

$$\eta \sim N(0, \sigma_\eta^2)$$

and the private signal x_i

$$x_i = \theta + \varepsilon_i$$

$$\varepsilon_i \sim N(0, \sigma_\varepsilon^2) \forall i$$

Solve for equilibrium average action \bar{a} as a function of θ and η .

- (2) Redo 1 but with only private signal. Is impact of θ larger or smaller?

7. GROSSMAN AND STIGLITZ AER (1980)

- (1) Describe the intuition behind Grossman and Stiglitz's Conjectures 1-6
- (2) Show analytically that prices convey more information when everybody is informed, compared to when nobody is informed.

8. ENDOGENOUS INFORMATION CHOICE AND RATIONAL INATTENTION

- (1) Describe intuitively what the "entropy" of a random variable mean.
- (2) Define mutual information
- (3) Formulate an information capacity constraint for a Gaussian signal Z and a Gaussian variable X .

- (4) Solve for the optimal allocation of attention (i.e. choose posterior variances) in the following set up:

Expected loss

$$EU = a\sigma_1^2 + b\sigma_2^2$$

subject to

$$\ln |\Sigma_{prior}| - \ln |\Sigma_{post}| \leq \kappa$$

where

$$\Sigma_{post} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

for $\Sigma_{prior} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $e^{-\kappa} = .5$, $a = b = 1$ and for $a = 1$ and $b = 0.01$.

Derive a relationship between a and b that ensures that the “no forgetting constraint” is not binding.

9. LEARNING AND BOUNDED RATIONALITY

- (1) Present an argument why it may be appealing to model agents as using RLS-learning.
- (2) Describe constant gain learning and in what environments it might make sense to use constant gain learning.
- (3) Explain or show mathematically why it is decreasing gain that put constant weight on all observations, while it is constant gain that puts decreasing weight on old observations.
- (4) Check if the system

$$p_t = \mu + \alpha E_{t-1} p_t + \delta w_{t-1} + \eta_t$$

$$p_t = a_{t-1} + b_{t-1} w_{t-1} + e_t \quad (PLM)$$

will converge to a REE equilibria.