TOPICS IN MACROECONOMICS: MODELLING INFORMATION, LEARNING AND EXPECTATIONS

EXERCISE QUESTIONS

These questions are representative of the questions that will be in the midterm. Essential reading for the course, in addition to the lecture notes, are Lucas AER (1973), Morris and Shin AER (2002), Grossman and Stiglitz, AER (1980).

1. Basic methods

(1) Consider the two random vectors X and Y

$$\begin{aligned} X &= \theta + u \\ y &= \theta + x \end{aligned}$$

with $E\left[\theta\theta'\right] = \sigma_\theta^2, E\left[uu'\right] = \sigma_u^2, E\left(u\theta'\right) = 0, E\left[xx'\right] = \sigma_x^2, E\left[x\theta'\right] = 0$ and $E\left[xu'\right] = 0$. Find the minimum variance linear estimator of X conditional on Y using the properties of orthogonal projections.

(2) Put the ARMA(1,2) process

$$y_t = A_1 y_{t-1} + C_0 \mathbf{u}_t + C_1 \mathbf{u}_{t-1} + C_2 \mathbf{u}_{t-2} \tag{1.1}$$

in the form

$$X_t = \mu_X + A X_{t-1} + C \mathbf{u}_t \tag{1.2}$$

(3) Derive an operational procedure for finding the autocovariance $\Sigma_{xx} \equiv E\left[X_t - \mu_X\right]\left[X_t - \mu_X\right]'$ for a system of the form (1.2). Be specific about additional assumptions made.

(4) Let $\mathcal{P}\left(X|Z,Y\right)$ denote the orthogonal projection of X on the space spanned by Z and Y. Show that

$$\mathcal{P}\left(X|Z,Y\right) = \mathcal{P}\left(X|Z\right) + \mathcal{P}\left(X|Y\right) \tag{1.3}$$

if $\mathcal{P}(Y|Z) = 0$.

2. The Kalman Filter

(1) For the state space system

$$X_t = AX_{t-1} + C\mathbf{u}_t \tag{2.1}$$

$$Z_t = DX_t + \mathbf{v}_t \tag{2.2}$$

$$\begin{bmatrix} \mathbf{u}_t \\ \mathbf{v}_t \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} CC' & \mathbf{0}_{n\times l} \\ \mathbf{0}_{l\times n} & \Sigma_{vv} \end{bmatrix}\right) \tag{2.3}$$

derive the first period Kalman gain K_1 in

$$X_{1|1} = AX_{0|0} + K_1\left(Z_1 - DAX_{0|0}\right) \tag{2.4}$$

taking $X_{0|0}$ and $P_{0|0} \equiv E\left(X_0 - X_{0|0}\right)\left(X_0 - X_{0|0}\right)'$ as given.

3. The Lucas AER (1973) Island model

(1) Interpret the slope parameter $\theta\gamma$ in the supply curve

$$y_t = \theta\gamma\left[P_t - \overline{P}_t\right] \tag{3.1}$$

the Lucas Island model.

(2) Explain the predictions made by the Lucas Island model regarding the correlation between cyclical output and inflation in different countries.

(3) Explain why information imperfections lead to higher output volatility in the Lucas Island model

4. Private and public information

(1) Consider the unobservable variable θ given by

$$\theta \sim N\left(0, \sigma_\theta^2\right) \tag{4.1}$$

Agents (indexed by j) observe a private noisy signal of θ given by

$$\begin{aligned} z(j) &= \theta + \varepsilon(j) \qquad (4.2) \\ \eta(j) &\sim N\left(0, \sigma_\varepsilon^2\right) \forall j \end{aligned}$$

That is, all agents receive an equally precise signal of θ but agent j only observes his own signal $z(j)$.Define

$$\begin{aligned} \theta^{(k)} &\equiv \int E\left[\theta^{(k-1)} \mid z(j)\right] \, dj \qquad (4.3) \\ \theta^{(0)} &\equiv \theta \qquad (4.4) \end{aligned}$$

Find an expression for $\theta^{(k)}$. What is the limit as $k \to \infty$?

(2) Redo 1) but with public signal

$$y = \theta + \eta$$

and θ non-stochastic (as in Morris and Shin 2002).

5. Morris and Shin AER (2002)

(1) Utility of agent i is given by

$$U_i = -\left(1 - r\right)\left(a_i - \theta\right)^2 - r\left(L_i - \overline{L}\right) \tag{5.1}$$

where a_i is the action taken by agent i and

$$L_i = \int (a_j - a_i)^2 \, dj \tag{5.2}$$

and

$$\overline{L} = \int L_j dj \tag{5.3}$$

Agents observe two signals of θ. The public signal y

$$\begin{aligned} y &= \theta + \eta \\ \eta &\sim N(0, \sigma_\eta^2) \end{aligned} \tag{5.4}$$

and the private signal x_i

$$\begin{aligned} x_i &= \theta + \varepsilon_i \\ \varepsilon_i &\sim N(0, \sigma_\varepsilon^2) \forall i \end{aligned} \tag{5.5}$$

Solve for equilibrium average action $\overline{a}$ as a function of θ and η.

(2) Redo 1 but with only private signal. Is impact of θ larger or smaller?

6. Grossman and Stiglitz AER (1980)

(1) Describe the intuition behind Grossman and Stiglitz's Conjectures 1-6
(2) Show analytically that prices convey more information when everybody is informed, compared to when nobody is informed.

7. Learning and Bounded Rationality

(1) Present an argument why it may be appealing to model agents as using RLS-learning.
(2) Describe constant gain learning and in what environments it might make sense to use constant gain learning.

(3) Explain or show mathematically why it is decreasing gain that put constant weight on all observations, while it is constant gain that puts decreasing weight on old observations.

(4) Check for E-stability of the system

$$p_t = \mu + \alpha E_{t-1} p_t + \delta w_{t-1} + \eta_t \tag{7.1}$$

$$p_t = a_{t-1} + b_{t-1} w_{t-1} + e_t \quad (PLM) \tag{7.2}$$