ECONOMETRIC METHODS II: TIME SERIES

HOMEWORK 2
DUE 17.00 TUESDAY JUNE 30

Instructions

Write up your results and submit electronically (preferably in pdf format) to knimark@crei.cat before 17.00 Tuesday June 30 together with any MatLab code used in the exercise. You may work in groups of up to three people. Please list all names in group on front page. To answer the questions, you will need to download the data file `HW2data.mat` from the course web page at http://www.kris-nimark.net/TS_UPF_2009.html .

1. A model

Consider the model

$$\begin{aligned}
x_t &= \rho x_{t-1} + u_t^x \\
y_t &= E_t(y_{t+1}) - \frac{1}{\gamma}\left[r_t - E_t\left(\pi_{t+1}\right)\right] \\
\pi_t &= E_t\left(\pi_{t+1}\right) + \kappa\left[y_t - x_t\right] + u_t^\pi \\
r_t &= \phi_y y_t + \phi_\pi \pi_t + u_t^r
\end{aligned}$$

Solving for output y_t and inflation π_t as function of potential output x_t and the shocks gives

$$\begin{aligned}
y_t &= -\kappa\frac{\rho - \phi_\pi}{c}x_t + \frac{1}{\gamma}u_t^r \\
\pi_t &= -\kappa\frac{\gamma + \phi_y - \gamma\rho}{c}x_t + u_t^\pi
\end{aligned}$$

where

$$c = \gamma + \phi_y - \kappa\rho - 2\gamma\rho + \kappa\phi_\pi - \rho\phi_y + \gamma\rho^2$$

The data file HW2data.mat contains

$$Z_t = \begin{bmatrix} y_1 & y_2 & \cdots & y_T \\ \pi_1 & \pi_2 & \cdots & \pi_T \\ r_1 & r_2 & \cdots & r_T \end{bmatrix}$$

Date: June 17, 2009.

Questions

(1) Find the state space representation of the model.

(2) Find the maximum likelihood estimate of the parameter vector $\theta = \{\rho, \gamma, \kappa, \phi_y, \phi_\pi, \sigma_x, \sigma_r, \sigma_\pi, \}$ using simulated annealing where σ_x, σ_r and σ_π are the standard deviations of the shocks u_t^x, u_t^r and u_t^π. Use $x_{0|0} = 0$ and $E\left(x_{0|0} - x_0\right)^2 = E\left(x_0^2\right)$ to initialize the Kalman filter.

(3) Estimate the posterior distributions of the parameters using the Random Walk Metropolis Algorithm. Report and motivate choices of initial values, covariance matrices, acceptance rates and upper and lower bounds of θ.
   (a) Plot the histograms of the simulated posterior distribution for each parameter. What do you conclude?
   (b) Check for convergence of the simulated posterior.
   (c) Find the impulse response function of interest rates r_t to a shock u_t^x to potential output. Plot the 95% probability intervals.