ECONOMETRIC METHODS II: TIME SERIES 2010

HOME WORK 2

Instructions

Write up your results and submit electronically (preferably in pdf format) to knimark@crei.cat **and** benedikt.herz@gmail.com before 24.00 Wednesday June 9 together with any MatLab code used in the exercise. You may work in groups of up to three people. Please list all names in group on front page. To answer the questions, you will need to download the data file `HW2data.zip` from the course web page at http://www.kris-nimark.net/TS_UPF_2010.html. The file `HW2data.zip` contains three data sets. Y1.mat and Y2.mat are both matrices containing 16 artificial data series with a sample length of 40. USdata.mat contains demeaned US inflation π and de-trended US GDP y in the form

$$\text{USdata.mat}=\begin{bmatrix} \pi_0 & \pi_1 & \cdots & \pi_T \\ y_0 & y_1 & \cdots & y_T \end{bmatrix}$$

The data is of quarterly frequency and the sample is from 1955:Q2-2010:Q1.

Question 1: Factor models and principal component

a) Use scree plots to determine whether Y1 and/or Y2 are likely to have been generated by an underlying factor structure. How many factors are necessary to capture "most" of the dynamics of Y1 and Y2 respectively?

b) Plot scree plots for the raw data, i.e. before normalizing the time series. What do you conclude?

c) For each data set that appear to be generated by a factor structure, plot the time series of the factors F_t that explain "most" of the variance of Y1 and or Y2.

d) Estimate a dynamic factor model of the form

$$F_t = AF_{t-1} + Cu_t : u_t \sim N(0, I) \tag{0.1}$$

for that/those data set(s) that appear to be explained by a factor structure.

e) Plot the impulse responses of the variables in the data set(s) that was/were generated by a factor structure to a shock to the first factor in (0.1).

Date: June 2, 2010.

Question 2: Potential output and the Kalman filter

Consider the basic New-Keynesian model

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \kappa(y_t - \overline{y}_t) + v_t^{\pi} : v_t^{\pi} \sim N(0, \sigma_{\pi}^2) \\
y_t &= E_t y_{t+1} - \gamma\left(i_t - E_t \pi_{t+1}\right) + v_t^{y} : v_t^{y} \sim N(0, \sigma_{y}^2) \\
i_t &= \phi \pi_t \\
\overline{y}_t &= \rho \overline{y}_{t-1} + u_t : u_t \sim N(0, \sigma_u^2)
\end{aligned}$$

When solved, the model can be put in state space form

$$\begin{aligned}
\overline{y}_t &= \rho \overline{y}_{t-1} + u_t \\
\pi_t &= \frac{\kappa\left(1-\rho\right)}{\Psi}\overline{y}_t + v_t^{\pi} \\
y_t &= \frac{\kappa\gamma\left(\rho-\phi\right)}{\Psi}\overline{y}_t + v_t^{y}
\end{aligned}$$

where $\Psi = \rho + \beta\rho - \beta\rho^2 - \kappa\gamma\phi + \kappa\gamma\rho - 1$. Unless otherwise indicated, use the following parameterization: $\left\{\beta, \kappa, \gamma, \phi, \rho, \sigma_{\pi}^2, \sigma_y^2, \sigma_u^2\right\} = \{.99, 0.2, 2, 1.5, 0.98, 1, 1, 0.01\}$ when answering the questions.

a) Put the model in the form

$$\begin{aligned}
X_t &= AX_{t-1} + C\mathbf{u}_t : \mathbf{u}_t \sim N(0, I) \\
Z_t &= DX_t + \mathbf{v}_t : \mathbf{v}_t \sim N(0, \Sigma_v)
\end{aligned}$$

that is, define the matrices A, C, D, Σ_v.

b) Use the Kalman filter to compute the time series $\left\{X_{t|t}\right\}_{t=1}^{T}$. Set the initial period mean and covariance to $X_{0|0} = \mathbf{0}$ and $P_{0|0} = E\left(X_t X_t'\right)$. (Notation as in lecture notes.)

c) Plot the time series $\left\{P_{t|t-1}\right\}_{t=1}^{T}$. Discuss. For $P_{0|0} = \mathbf{0}$, plot $\left\{X_{t|t}\right\}_{t=1}^{T}$ and $\left\{P_{t|t-1}\right\}_{t=1}^{T}$. Discuss.

d) Set $P_{0|0} = E\left(X_t X_t'\right)$ and $\left\{\sigma_{\pi}^2, \sigma_y^2\right\} = \{1000000, 1000000\}$. Plot the time series $\left\{P_{t|t-1}\right\}_{t=1}^{T}$. Discuss.

e) For the original parameterization, plot potential output $\overline{y}_t$ and output y_t in the same graph. What is the sample variance of the output gap $y_t - \overline{y}_t$?

f) Repeat e) for $\sigma_u^2 = 1$. Discuss.