ECONOMETRIC METHODS II: TIME SERIES

SPRING 2011
HOMEWORK 4

Instructions

This homework tackles SVARs from a Bayesian perspective and the instructions are thus similar to those of Homework II. Choose a country for your group (it can be the same country as in HW II) and find time series for (i) an official (short) interest rate (r_t), (ii) a measure of CPI (or similar) and (iii) real GDP. As you know by now, a good place to look for data is the IMF web page www.imfstatistics.org or individual countries' central bank web pages. Choose a country for which at least 40 years of data is available (i.e. so that $T > 120$). **Make sure that the data is seasonally adjusted, or seasonally adjust it your self.**

Write up your results and submit electronically (preferably in pdf format) to knimark@crei.cat and fernandojose.perez@upf.edu before **13.00 Thursday June 16** together with any MatLab code used in the exercise. Explain carefully each step involved in answering each question. You may work in groups of up to four people. Please list all names in group on front page.

Instructions for in-class presentations: The exercise will be presented in class on Thursday June 16. For the presentations, each group should prepare slides that can be displayed using the computer projector and choose a spokes person. Each group has 10 minutes to explain what they did and what they found. Bring the slides for the presentation on a USB memory.

Date: June 10, 2011.

A Bayesian perspective on SVARs

a) Transform GDP and CPI into growth rates (Δy_t) and inflation rates (π_t) respectively, as follows $\Delta y_t = \ln(GDP_t) - \ln(GDP_{t-1})$ and $\pi_t = \ln(CPI_t) - \ln(CPI_{t-1})$. Demean the time series and plot the transformed data.

b) Consider the trivariate reduced form VAR(1) model

$$\begin{bmatrix} r_t \\ \Delta y_t \\ \pi_t \end{bmatrix} = \phi \begin{bmatrix} r_{t-1} \\ \Delta y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \mathbf{e}_t : \mathbf{e}_t \sim N(0, \Omega) \tag{0.1}$$

What is the vector of parameters θ? What is the dimension of θ?

c) Estimate the posterior distribution of θ of the reduced form VAR(1) model from part b) using the Random-Walk Metropolis-Hastings Algorithm with uniform (truncated) priors. **Tip:** The log likelihood of a VAR is given by

$$\begin{aligned} \ln L(y^T \mid \theta) = -0.5 \sum_{s=2}^{T} \left(p \ln(2\pi) + \ln|\Omega| + \mathbf{e}_s' \Omega^{-1} \mathbf{e}_s \right) \\ -0.5 \left(p \ln(2\pi) + \ln|\Sigma_{yy}| + y_1' \Sigma_{yy}^{-1} y_1 \right) \end{aligned} \tag{0.2}$$

where

$$\Sigma_{yy} \equiv E\left(y_t - \mu_y\right)\left(y_t - \mu_y\right)' \tag{0.3}$$

and p is the dimension of y_t. Plot the time series and the histograms of the Markov Chain representing the estimated posterior distribution of θ. How many draws do you need for the Markov Chain to converge? What is the acceptance rate in the M-H algorithm?

d) Estimate the posterior distribution of θ of the unrestricted structural VAR(1)

$$A_0 \begin{bmatrix} r_t \\ \Delta y_t \\ \pi_t \end{bmatrix} = A_1 \begin{bmatrix} r_{t-1} \\ \Delta y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \mathbf{u}_t : \mathbf{u}_t \sim N(0, I)$$

using the Metropolis-Hastings Algorithm with uniform (truncated) priors. What is the parameter vector θ? Plot the histograms of the posteriors for θ. What do you conclude?

e) Re-estimate the posterior distribution for θ from the SVAR in d) imposing that A_0 is lower triangular. Plot the histograms of the posteriors for θ. What do you conclude? Construct posterior 95% probability intervals for the impulse responses of all variables to an interest rate shock. Find the posterior 95% probability intervals for the decomposition of individual variances into the orthogonal innovations $\mathbf{u}_t$.

f) Re-estimate the posterior distribution for θ from the SVAR in d) imposing that A_0 is diagonal and that A_1 is lower triangular. What is θ? Plot the histograms of θ. Construct posterior 95% probability intervals for the impulse responses of all variables to an interest rate shock. What do you conclude?