INDICATOR ACCURACY, MONETARY POLICY AND WELFARE

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Abstract. We present a New-Keynesian model with optimal discretionary monetary policy, where households and the central bank have partial and diverse information. This set up allow us to separate the welfare effects of having a better informed central bank versus better informed households. The model is used to show that better informed households respond stronger to cost-push shocks which in turn leads to larger relative price distortions due to staggered prices. This implies that the representative household is better off when households (as a class of agents) have less accurate information. Improving the precision of the central banks' information leads to a more accurate trade-off between inflation and output and increases welfare. Assuming a common information set shared by households and the central bank is thus inappropriate when one is concerned with the welfare effects of information itself.

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1. Introduction

Recent advances in modelling monetary policy have allowed economists to study how monetary policy is affected by noisy indicators. Building on earlier work on control with partial information by Pearlman (1986) and Currie, Levine and Pearlman (1986), Svensson and Woodford (2003, 2004) provide general procedures to solve a class of monetary policy models where some variables are unobservable and some are only observable with noise. Their framework has been utilized by for instance Ehrmann and Smets (2001) who investigate the performance of different policy rules in a calibrated forward/backward looking model and Coenen, Levin and Wieland (2002) who quantify the usefulness of monetary aggregates as indicator variables. Lippi and Neri (2003) contribute to related empirical methods by showing how the indicator accuracy and the structural parameters of a model can be estimated simultaneously. As noted by Lippi and Neri (2003) and Nimark (2003) noisy indicators often improve macroeconomic outcomes in this class of models, and thus suggests that central bankers do not benefit from the availability of accurate real time data. This paper argues that this result is strongly dependent on the

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assumption of a *common* information set shared by the central bank and the representative household. In the model presented below, the central bank and the representative household are endowed with *diverse* (but intersecting) information sets. This allow us to separate the welfare effects of having a better informed central bank versus a better informed public.

In the presence of shocks that create a trade off between stabilizing inflation and output, we show that welfare is decreasing in the precision of the information set of the representative household. The intuition behind the result is that more uncertainty on the behalf of households induce weaker responses to shocks and therefore present the policy makers with a more favorable trade off. The welfare effects of changing the precision of the central bank's information set are a priori ambiguous. The reason for the ambiguity is that while less precise information prevents the central bank from optimally trading off inflation and output deviations, it also makes policy more inertial. To the degree that the more inertial character of policy is anticipated by the public, less precise information can improve welfare by decreasing the so-called stabilization bias of discretionary policy. Numerical simulations suggest that the negative welfare effect of less precise central bank information dominates.

The diverse information structure presents us with two modelling challenges. In the structural model, households' expectations about future inflation and output partly determine inflation and output today. These expectations are not observable by the central bank but relevant for the optimal setting of the interest rate. We use a method similar to Sargent (1991) to model the central bank's estimate of the unobservable household expectations. Secondly, when households and the central bank have diverse information, policy makers will make serially correlated 'policy mistakes' that are partly detectable by the households. We show how this information can be used by households to make more efficient forecasts of future endogenous variables. Aoki (2003) and Svensson and Woodford (2004), like the present paper, set up a model of diversity of information between the central bank and the public. However, in their framework the public is perfectly informed which simplifies the filtering problem of the central bank.

The next section presents the structural model. Section 3 uses a simple example to analytically show why the variance of inflation and the output gap is decreasing in the variance of the noise in the indicators available to households. Section 4 presents the diverse information structure that is used in the rest of the paper. Section 5 and 6 presents the decision and filtering problems of the central bank and households respectively. Section 7 contains numerical results on the welfare effects of varying the precision of the indicators available to the central bank and households as well as a discussion on how the increase in welfare from less informed households should be interpreted at the micro level. Section 8 concludes.

2. A business cycle model

This section presents a standard New Keynesian business cycle model with monopolistically competitive firms that sell differentiated goods and where pricesetting is restricted by the Calvo (1983) mechanism. The exposition follows Gali (2002) quite closely. In what follows, lower case letters denote log deviations from steady state values of the variable denoted by the corresponding capital letter.

2.1. **Households and Firms.** Consider a representative household that wishes to maximize the discounted sum of expected utility

$$E\left\{\sum_{s=0}^{\infty}\beta^{s}U\left(C_{t+s},N_{t+s}\right)\mid I_{t}^{h})\right\} \tag{2.1}$$

where $\beta \in (0,1)$ is the household's discount factor and I_t^h is the information set available to households in period t and defined in the next section. The period utility function in consumption C_t and labor N_t is given by

$$U\left(C_t,N_t\right)=\frac{{C_t}^{1-\gamma}}{(1-\gamma)}-\frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{2.2}$$

where C_t is the standard CES aggregator

$$C_t=\left(\int_0^1 C_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2.3}$$

The optimal demand for individual goods indexed by $j \in (0,1)$ then is

$$c_t(j)=-\epsilon(p_t(j)-p_t)+c_t \tag{2.4}$$

Good j is produced by firm j with a technology that is linear in labor and subject to a persistent technology shocks A_t

$$Y_t(j)=A_tN(j). \tag{2.5}$$

Labor supply enters the utility function (2.2) only in the aggregate as $N_t \equiv \int N_t(j)$ dj. The government purchases a fraction τ_t of all goods produced in the economy. The demand for good j (in logs) can then be written as

$$y_t(j)=c_t(j)+g_t \tag{2.6}$$

where $g_t=-\log\left(1-\tau_t\right)$. There is no storage technology and clearing of all goods markets imply

$$y_t=c_t+g_t \tag{2.7}$$

where $y_t=\log\left(\int_0^1 Y_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

2.2. **Optimality conditions.** Households decide how much to consume and how much labor to supply. Intertemporal optimization of consumption yields the standard Euler equation

$$c_t=E_tc_{t+1}-\frac{1}{\gamma}\left[i_t-E_t\pi_{t+1}\right] \tag{2.8}$$

where i_t and π_t are the nominal interest rate and inflation, respectively. Substitute in the resource constraint $y_t=c_t+g_t$ to get

$$y_t=E_ty_{t+1}-\frac{1}{\gamma}\left[i_t-E_t\pi_{t+1}\right]+\left(1-\rho_g\right)g_t \tag{2.9}$$

The optimal labor supply can be found by equating the marginal disutility of supplying labor with the marginal utility of consumption times the real wage

$$-\gamma c_t+w_t-p_t=\varphi n_t \tag{2.10}$$

where w_t-p_t is the real wage in period t. Using again the resource constraint (2.6) and that $y_t=n_t+a_t$ we get

$$-\gamma(y_t - g_t) + w_t - p_t = \varphi\,(y_t - a_t) \tag{2.11}$$

which we can rearrange to

$$w_t - p_t = (\gamma + \varphi)\,y_t - \varphi a_t - \gamma g_t. \tag{2.12}$$

Real marginal cost then equals the real wage divided by the productivity of labor

$$mc_t = (\gamma + \varphi)\,y_t - (1 + \varphi)a_t - \gamma g_t \tag{2.13}$$

The natural, or potential, level of output $\overline{y}_t$, defined as the level of output that would prevail under flexible prices coincides with the the level that is consistent with no inflation. It can be found by setting $mc_t = 0$ in (2.13) and solving for the output level

$$\overline{y}_t = \frac{(1+\varphi)}{(\gamma+\varphi)}a_t + \frac{\gamma}{(\gamma+\varphi)}g_t \tag{2.14}$$

2.3. **Price setting.** As in Calvo (1983), a fraction $(1 - \theta)$ of firms adjust prices each period and the price level thus follows

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{2.15}$$

where

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[p_{t+k} + mc_{t+k}\right] \tag{2.16}$$

is the optimal price set by firms adjusting prices in period t. The derivation of the firms' price setting problem can be found in Appendix A. Rewriting (2.16) as

$$p_t^* = (1-\beta\theta)\left[p_t + mc_t\right] + \beta\theta E_t p_{t+1}^* \tag{2.17}$$

and substituting it into the expression for the price level (2.15) gives

$$p_t = \theta p_{t-1} + (1-\theta)(1-\beta\theta)\left[p_t + mc_t\right] + (1-\theta)\beta\theta E_t p_{t+1}^*$$

Rearranging and using that

$$\begin{aligned} \pi_{t+1} &= p_t - p_{t-1} \\ &= (1-\theta)p_{t+1}^* + (\theta - 1)\,p_t \end{aligned} \tag{2.18}$$

we get the New Keynesian Phillips curve

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}mc_t + \beta E_t\pi_{t+1} + \varepsilon_t \tag{2.19}$$

or in terms of the output gap

$$\pi_t = \delta\phi(y_t - \overline{y}_t) + \beta E_t\pi_{t+1} + \varepsilon_t \tag{2.20}$$

where

$$\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \quad \phi \equiv (\gamma + \varphi)$$

The "cost push" shock $\varepsilon_{\pi t}$ can be interpreted as a shock to mark ups.[1]

[1] See for instance Smets and Wouters (2003).

2.4. **Collecting equations.** The exogenous shocks $\{\varepsilon_t, g_t, a_t\}$ all follow AR(1) processes. For ease of notation we can collect them in the vector $X_t = [\varepsilon_t \; g_t \; a_t]'$

$$X_t = \rho X_{t-1} + \mathbf{u}_t \tag{2.21}$$

where

$$\rho = \begin{bmatrix} \rho_\varepsilon & 0 & 0 \\ 0 & \rho_g & 0 \\ 0 & 0 & \rho_a \end{bmatrix} \tag{2.22}$$

$$\mathbf{u}_t \sim N(\mathbf{0},\Sigma_{uu}), \quad \Sigma_{uu} = \begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 \\ 0 & \sigma_g^2 & 0 \\ 0 & 0 & \sigma_a^2 \end{bmatrix}$$

Equations (2.9) and (2.20) describe the dynamics of the endogenous variables inflation and output, and can be put in compact form as

$$x_t = AX_t + CE_t x_{t+1} + Bi_t, \tag{2.23}$$

where $x_t = [\pi_t \;\; y_t]'$ and the matrices A, B and C are defined as

$$A \equiv A_0^{-1}A_1, C \equiv A_0^{-1}C_1, B \equiv A_0^{-1}B_1. \tag{2.24}$$

$$A_0 = \begin{bmatrix} 1 & -\delta\phi \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & -\delta(1+\varphi) & -\delta\gamma \\ 0 & (1-\rho_g) & 0 \end{bmatrix}, \tag{2.25}$$

$$C_1 = \begin{bmatrix} \beta & 0 \\ \frac{1}{\gamma} & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -\frac{1}{\gamma} \end{bmatrix}. \tag{2.26}$$

The system can be compactly represented by (2.21) and (2.23).

2.5. **The Policymaker.** The central bank sets the interest rate to minimize the loss function

$$\Lambda_t = E\left[\sum_{k=0}^{\infty} \beta^k \left[\lambda(y_{t+k} - \overline{y}_{t+k})^2 + \pi_{t+k}^2\right] \mid I_t^{cb}\right] \tag{2.27}$$

where λ is a preference parameter of the central bank and I_t^{cb} is the information set of the central bank at time t, which we define below. Optimal policy under full information will be characterized by the central bank's first order condition

$$y_t - \overline{y}_t = -\frac{\phi\delta}{\lambda}\pi_t \tag{2.28}$$

which can be found by solving the Lagrangian problem consisting of (2.27) as the objective function and the Phillips curve (2.20) as the constraint. The central bank's preference parameter λ is chosen to equal $\frac{\delta\phi}{\epsilon}$ which implies that the objective of the central bank coincides with maximizing the utility of the representative household.[2] The first order condition (2.28) can then be written simply as

$$y_t - \overline{y}_t = -\epsilon\pi_t \tag{2.29}$$

The New-Keynesian Phillips curve (2.20) and a $\lambda > 0$ in (2.27) imply that the central bank faces a trade off between inflation and output gap stabilization in the presence of cost-push shocks. Optimal policy can offset demand and technology shocks completely and a perfectly informed central bank thus only suffers losses from the cost-push shocks.

[2] See Woodford (1999) for a derivation of the lossfunction as a second order approximation of the representative household's utilityfunction.

3. Households' information and the variance of the target variables

In the presence of frictions or externalities, the optimal decisions of individuals are not necessarily optimal from a social perspective. In the present model with staggered prices, individually optimal pricing decisions of firms lead to socially inefficient inflation. We argue below that these externalities also have implications for what the socially optimal precision of information is and that households as a class of agents are better off with less precise information. We will make the case in several steps. In this section we use a simple set up with a single shock affecting the economy and a perfectly informed central bank to illustrate the channels through which noise in the households' indicator reduce the variance of inflation and the output gap. The simple example is meant to aid intuition and it is followed by a more realistic, but inevitably more complex, information structure where the central bank and households have partial and diverse (but intersecting) information sets. Through numerical simulations we show that the qualitative results from the simple model carries over to the more complex setting. We also investigate numerically the effects of changing the precision of the indicators available to the central bank, as well as to the common indicators available to both the central bank and households.

3.1. **Household estimates and the target variables.** In this section we want to isolate the effects of changing the accuracy of the households' private indicators and we temporarily assume a perfectly informed central bank. This allows us substitute the first order condition (2.28) into the Phillips curve (2.20) to get

$$\pi_t = \beta\pi^h_{t+1|t} - \delta\phi\epsilon\pi_t + \varepsilon_t. \tag{3.1}$$

where

$$\pi^h_{t+1|t} = E\left[\pi_{t+1} \mid I^h_t\right]$$

and I^h_t is the information set available to households at time t. Rearrange (3.1) to get inflation as a function of the cost-push shock and expected future inflation

$$\pi_t = \frac{\beta}{1+\delta\phi\epsilon}\pi^h_{t+1|t} + \frac{1}{1+\delta\phi\epsilon}\varepsilon_t. \tag{3.2}$$

Iterate forward to eliminate the inflation expectation

$$\pi_t = \frac{1}{1+\delta\phi\epsilon}\sum_{k=1}^{\infty}\left(\frac{\beta}{1+\delta\phi\epsilon}\rho_\varepsilon\right)^k \varepsilon^h_{t|t} + \frac{1}{1+\delta\phi\epsilon}\varepsilon_t \tag{3.3}$$

where we used that $E\left[\varepsilon_{t+k} \mid I^h_t\right] = \rho^k_\varepsilon E\left[\varepsilon_t \mid I^h_t\right]$. That

$$0 < \frac{\beta}{1+\delta\phi\epsilon}\rho_\varepsilon < 1 \tag{3.4}$$

ensures that the sum is finite. Replace the the cost push shock in the forward looking sum with the household estimate and redefine the coefficients as

$$\chi \equiv \frac{1}{1+\delta\phi\epsilon}\sum_{k=1}^{\infty}\left(\frac{\beta}{1+\delta\phi\epsilon}\rho_\varepsilon\right)^k,\ \omega \equiv \frac{1}{1+\delta\phi\epsilon}.$$

The variance of inflation, σ^2_π, can then be written as

$$\sigma^2_\pi = \chi^2\sigma^{2h}_\varepsilon + \omega^2\sigma^2_\varepsilon + 2E\left[\omega\chi\varepsilon_t\varepsilon^h_{t|t}\right] \tag{3.5}$$

where σ_ε^2 and σ_ε^{2h} are the variances of the actual and estimate of the cost-push shocks respectively. We thus need to show that the variance of the estimate as well as the covariance of the estimate and the actual shock is decreasing in the variance of the measurement errors. With a perfectly informed central bank, this means that the variance of the output gap is also decreasing (and welfare unambiguously increasing) since the first order condition imply that

$$\sigma_{y-\overline{y}}^2 = \epsilon^2 \sigma_\pi^2 \tag{3.6}$$

holds exactly. The role played by expectations about the future can also be made clear if we note that (3.5) shows that the variance of both inflation and output are increasing in ρ and β. If either today tells us little about tomorrow, i.e. if ρ_ε is small, or if households don't care much for tomorrow, i.e. if β is small, the variance of the estimates will have a small impact on the variance of inflation and the output gap. We now turn to the question of how the variance of the estimate, σ_ε^{2h}, is affected by less accurate household indicators.

3.2. **Noisy indicators and the variance of the estimates.** In this simple example, households observe a direct but noisy measure of the cost push shock. The cost push shock is the only structural shock affecting the system and follows an AR(1) process as in (2.21). Households estimate the cost push shock ε_t recursively by applying the Kalman filter to the noisy observation z_t and the estimate of the shock is given by the updating equation

$$\varepsilon_{t|t}^h = (1-k)\,\rho_\varepsilon \varepsilon_{t-1|t-1}^h + k z_t \tag{3.7}$$

where

$$\begin{aligned} z_t &= \varepsilon_t + v_t && (3.8)\\ \varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + u_{\varepsilon t} && (3.9)\\ \begin{bmatrix} v_t \\ u_{\varepsilon t} \end{bmatrix} &\sim N\left(\mathbf{0}, \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix}\right) && (3.10) \end{aligned}$$

The optimal gain k in (3.7) is then given by

$$k = \frac{p}{p+\sigma_v^2} \tag{3.11}$$

where p satisfies the Riccati equation

$$p = \rho_\varepsilon^2 p - \frac{(\rho_\varepsilon p)^2}{p+\sigma_v^2} + \sigma_\varepsilon^2 \tag{3.12}$$

In (3.7) the current estimate $\varepsilon_{t|t}^h$ is a weighted average of the pre-observation prior $\rho_\varepsilon \varepsilon_{t-1|t-1}^h$ and the observation z_t with the weights determined by the relative variance of the innovation to the cost-push shock u_t, the measurement error v_t and the persistence of the cost-push shock ρ_ε.[3] In the figure below, simulated estimates of the cost push shocks are plotted for different magnitudes of measurement errors together with the actual cost-push shocks hitting the economy.

[3]See Maybeck (1979) for an introduction to the Kalman filter.

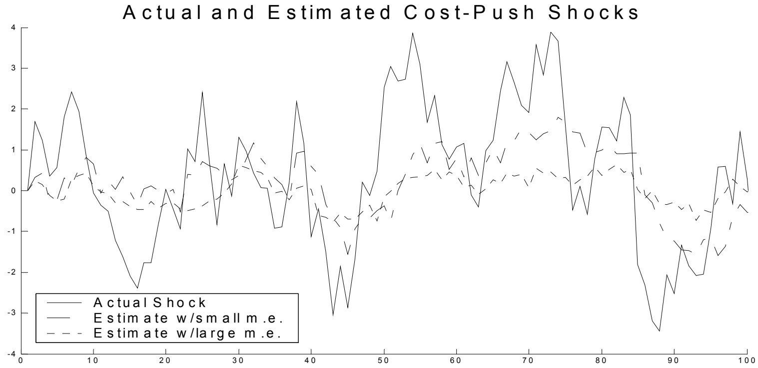

Figure 1

We can see that the larger the measurement errors, the closer to the (zero) mean are the estimates. To understand the mechanics of this, it is instructive to look at the limit case of infinitely noisy indicators. It is clear from (3.7) and (3.11) that when the measurement error variance σ_v^2 increases, households put less weight on the observation z_t and in the limit case of an infinitely noisy measure, the weight tends to zero. In this case the observation contains no information at all about the actual cost push shock and the updating equation (3.7) will converge towards the unconditional (zero) mean of the cost push shock from any initial value. An infinitely noisy observation thus also imply that the covariance of the estimate and the actual shock will be zero. Of course, in the opposite case, when the noise tends to zero, the two variables co-vary perfectly since it implies that $z_t = \varepsilon_t$, i.e. the actual shock is perfectly observed and k tends to 1. The effect of decreased variance and covariance of the estimates of the cost-push shocks are illustrated in Figure 2 where we have plotted the inflation under perfectly observed cost push shocks as well as when the shocks are only observed with noise for the same realization of shocks as in Figure 1.

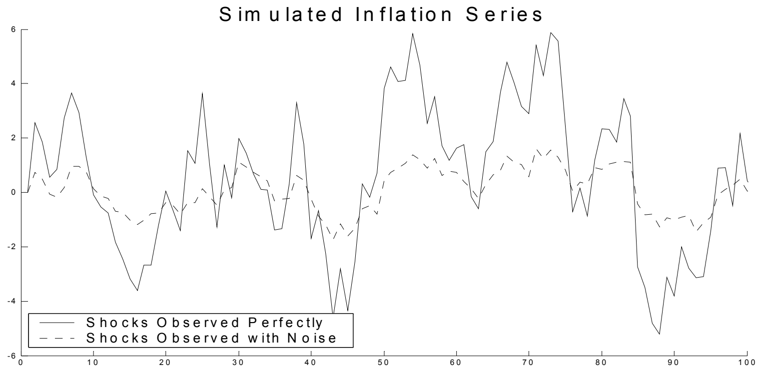

Figure 2

While it is intuitive that agents put less weight on noisy observations, this fact by itself is not enough for the variance of the estimate to be decreasing in the variance of the noise. The reason is that when the variance of the noise increases, then so does the variance of the observation. We thus need to show that the weight on the observation is decreasing "faster" than the variance of the observation is increasing in the magnitude of the noise. The variance of the estimate as a function of the actual shock variance σ_{ε}^{2}, the measurement error variance σ_{v}^{2}, the persistence parameter ρ_{ε} and the Kalman gain k is given by

$$\sigma_{\varepsilon}^{2h}=\frac{2k\rho\sigma_{\varepsilon\varepsilon h}+k^{2}\left(\frac{\sigma_{\varepsilon}^{2}}{1-\rho_{\varepsilon}^{2}}+\sigma_{v}^{2}+\sigma_{e}^{2}-2\rho\sigma_{\varepsilon\varepsilon h}\right)}{\left(1-\rho^{2}+2k\rho^{2}-k^{2}\rho^{2}\right)} \tag{3.13}$$

where $\sigma_{\varepsilon\varepsilon h}$ is the covariance between the estimate and the actual shock

$$\sigma_{\varepsilon\varepsilon h}=\frac{k\sigma_{\varepsilon}^{2}}{\left(1-\rho^{2}+k\rho^{2}\right)}+\frac{\sigma_{\varepsilon}^{2}kp}{\left(1-\rho^{2}\right)\left(1-\rho^{2}+k\rho^{2}\right)}. \tag{3.14}$$

The expressions (3.13) and (3.14) are derived in the Appendix and direct computation confirms that both (3.13) and (3.14) are decreasing in the variance of the measurement errors σ_{v}^{2}. This completes the link from household indicator noise to the variance of the target variables. Less informed households thus tend to reduce the losses suffered from inflation and output gap volatility.

The analytical results of this section were derived under the special assumptions of a perfectly informed central bank and cost push shocks as the only source of disturbances in the economy. The next section presents a more realistic information set up where neither the central bank nor households are perfectly informed.

4. Diversity of Information

In full information models it is assumed that all agents know the complete structure of the economy and can observe all relevant variables perfectly. The full information set at time T shared by all agents, I_{T}^{f}, thus is

$$I_{T}^{f}=\left\{A,T,B,\lambda,X_{t},x_{t},u_{t},\Sigma_{\mathrm{uu}}\mid t\leq T\right\} \tag{4.1}$$

We depart from this setting by making the exogenous shocks, X_{t}, unobservable and the endogenous variables x_{t} observable only with measurement error. We also make a distinction between observations available to the central bank and those available to the representative household. The information set of the partially informed agent i at time T, I_{T}^{i}, is defined by (4.2)

$$I_{T}^{i}=\left\{A,T,D,D^{i},\lambda,\Sigma_{\mathrm{uu}},\Sigma_{vv}^{i},Z_{t}^{i}\mid t\leq T\right\},i\in\{cb,h\} \tag{4.2}$$

$$Z_{t} = D\left[\begin{array}{c}X_{t}\\ x_{t}\end{array}\right]+\mathbf{v}_{t} \tag{4.3}$$

$$Z_{t}^{i} = D^{i}Z_{t} \tag{4.4}$$

where D is a matrix that picks out and scales the variables Z_{t} that are observable *in principle*, while D^{i} picks out what variables that are observable to agent i. The superscript cb denotes the central bank's information set while h denotes that of households. Specifically,

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad Z_t = \begin{bmatrix} \pi_t \\ y_t \\ y_t \\ y_t \end{bmatrix} + \mathbf{v}_t$$

$$\mathbf{v}_t = \begin{bmatrix} v_{t\pi} \\ v_{ty}^1 \\ v_{ty}^2 \\ v_{ty}^3 \end{bmatrix}, \quad \mathbf{v}_t \sim N\left(\mathbf{0}, \Sigma_{vv}\right)$$

The diversity of information will be modelled by allowing the central bank to observe the inflation measure and the *first* two measures of output, while households will observe the inflation measure and the *last* two measures of output. The measure of inflation and the second measure of output are thus the commonly observed indicators. Formally

$$D^{cb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad Z_t^{cb} = \begin{bmatrix} \pi_t \\ y_t \\ y_t \end{bmatrix} + \mathbf{v}_t^{cb},$$

$$\mathbf{v}_t^{cb} = \begin{bmatrix} v_{t\pi} \\ v_{ty}^1 \\ v_{ty}^2 \end{bmatrix}, \quad \mathbf{v}_t^{cb} \sim N\left(\mathbf{0}, \Sigma_{vv}^{cb}\right)$$

$$D^{h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Z_t^{h} = \begin{bmatrix} \pi_t \\ y_t \\ y_t \end{bmatrix} + \mathbf{v}_t^{h}$$

$$\mathbf{v}_t^{h} = \begin{bmatrix} v_{t\pi} \\ v_{ty}^2 \\ v_{ty}^3 \end{bmatrix}, \quad \mathbf{v}_t^{h} \sim N\left(\mathbf{0}, \Sigma_{vv}^{h}\right).$$

Varying the accuracy of the private indicator of the central bank can be implemented by changing the variance of v_{ty}^1. Similarly, the accuracy of households private information can be varied by changing the variance of v_{ty}^3. It then follows that the precision of the common information is captured by the variance of $v_{t\pi}$ and v_{ty}^2. Some comments on this information structure is in order. Assuming only one common measure of inflation is intended to capture the fact that inflation is measured quickly and accurately and that the numbers are publicly available almost immediately. Letting the central bank and the public have diverse but intersecting indicator sets for output is meant to capture that a lot of the information about the level of activity in the economy is publicly available, but that the central bank may have some private information, perhaps from survey data etc., that is not published. We treat households as an "informational monolith" that all possess the same information. This simplifying assumption keeps the model tractable but raises questions about the interpretation of the indicators that are observable to *all* of the households but not to the central bank. The assumption of households having identical information should not be taken literally, but rather as a tractable way of modelling diversity of information between the public and the the central bank without introducing the additional complications that arise when households and firms have diverse information with respect to each other. Such models are analyzed by for instance Woodford (2002), Amato and Shin (2004) and Nimark (2005), and the results derived here should be robust to the presence of diversity of information within the households of the economy. Instead, we interpret the information set common to all households as a "representative" information set conceptually similar to the consumption or labor supply of the representative household, and there is no obvious reason to believe that this representative information set is a subset of the information set of the central bank.

5. Optimal policy and diverse information

The central bank's problem is to minimize

$$\Lambda_t = E\left[\sum_{k=0}^{\infty} \beta^k \left[\lambda(y_{t+k} - \overline{y}_{t+k})^2 + \pi_{t+k}^2\right] \mid I_t^{cb}\right] \tag{5.1}$$

subject to the structural equation

$$x_t = AX_t + Cx_{t+1|t}^h + Bi_t \tag{5.2}$$

where $x_t = [\pi_t \ \ y_t]'$. Since the central bank and households do not share the same information set the central bank does not know the expectations of households with certainty. To minimize (5.1) the central bank need to estimate both the exogenous process X_t as well as the impact of households' expectations on inflation and the output gap. Neither of these are directly observable. While the exogenous process X_t is independent of the measurement error in the model, the stochastic properties of households' expectations depend on the precision of the information available to households as well as on the precision of the information available to the central bank. To see why, one should note that households' expectations of future endogenous variables depend on both expectations about the exogenous variables as well as expectations about future policy, which in turn depend on the information available to the central bank. We use a method similar to Sargent (1991) to model the interaction of expectations and information sets. The strategy is to define a joint stochastic process for the unobservable exogenous state and the impact of households' expectations. The central bank then uses the Kalman filter to estimate the current state of the economy, where the shape of the Kalman filter will depend on the implied joint equilibrium dynamics of the exogenous state and the expectations of households.

Define the residual variable ζ_t

$$\zeta_t \equiv x_t - AX_t - Bi_t \tag{5.3}$$

It then follows from the structural equation that

$$\zeta_t = Cx_{t+1|t}^h \tag{5.4}$$

i.e. the residual variable ζ_t captures the impact of the actions of households on the observable endogenous variables. The central bank uses the Kalman filter to estimate the extended state $\widehat{X}_t$ defined as

$$\widehat{X}_t \equiv [X_t \ \ \zeta_t \ ]' \tag{5.5}$$

with the updating equation

$$\widehat{X}_{t|t}^{cb} = \widehat{X}_{t|t-1}^{cb} + K^{cb}\left[Z_t^{cb} - W\left(\widehat{L}\widehat{X}_{t|t-1}^{cb} + Bi_t\right)\right] \tag{5.6}$$

The term in square brackets is the "surprise" component, or innovation, in the central bank's observation Z_t^{cb}. The central bank uses a linear projection of $\widehat{X}_t^{cb}$ on X_{t-1} to form a period t pre-observation prior of the extended state $\widehat{X}_t^{cb}$. The prior $\widehat{X}_{t|t-1}^{cb}$ is then given by

$$\widehat{X}_{t|t-1}^{cb} = \widehat{\boldsymbol{\rho}}\widehat{X}_{t-1|t-1}^{cb} \tag{5.7}$$

$$\widehat{\boldsymbol{\rho}} = \begin{bmatrix} \boldsymbol{\rho} & 0 \\ \Sigma_{\zeta X_{t-1}}\Sigma_{XX}^{-1} & 0 \end{bmatrix} \tag{5.8}$$

where

$$\Sigma_{\zeta X_{t-1}} = E\left[\begin{pmatrix} \zeta_t & X_{t-1} \end{pmatrix}\begin{pmatrix} \zeta_t & X_{t-1} \end{pmatrix}'\right] \tag{5.9}$$

$$\Sigma_{XX} = E\left[X_t X_t'\right] \tag{5.10}$$

We also need to define the innovation component of ζ_t

$$\psi_t \equiv \zeta_t - \Sigma_{\zeta X_{t-1}}\Sigma_{XX}^{-1}X_{t-1} \tag{5.11}$$

$$\Sigma_{\psi\psi} = E\left(\psi_t\psi_t'\right) \tag{5.12}$$

i.e. ψ_t is the difference between the realization of ζ_t and the pre-observation expectation conditional on the previous period exogenous state. The central bank's Kalman gain vector K^{cb} can be computed using (2.21), (5.8), (5.11) and (5.12). The Kalman filter formulas and the definitions of the matrices W and $\widehat{L}$ are given in the Appendix.

The central bank's problem is thus to set the short interest rate to minimize (5.1) given the estimate of the current extended state $\widehat{X}_{t|t}^{cb}$. With a quadratic objective function and linear constraints we know that the optimal policy is certainty equivalent and thus independent of the variance of the shocks of the economy. We can combine the loss function (5.1) and the constraint (5.2)

$$\Lambda_t = E\left[\sum_{k=0}^{\infty}\beta^k \widehat{X}_t' Q' R Q \widehat{X}_t \mid I_t^{cb}\right] \tag{5.13}$$

where the matrices R and Q are defined as

$$R \equiv \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}, \quad Q \equiv N + BF \tag{5.14}$$

$$N = \widehat{L} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\gamma}{(\gamma+\varphi)} & \frac{(1+\varphi)}{(\gamma+\varphi)} & 0 & 0 \end{bmatrix} \tag{5.15}$$

The optimal interest function

$$i_t = F\widehat{X}_{t|t}^{cb} \tag{5.16}$$

is yet to be determined. There is no endogenous persistence in the model since the state is exogenous, so minimizing the sum (5.13) under discretion is equivalent to minimizing the loss function period by period. Taking derivatives of (5.13) at time t with respect to F yields

$$\frac{\partial \Lambda_{t|t}^{cb}}{\partial F} = 2B'RN\widehat{X}_{t|t}\widehat{X}_{t|t}' + 2B'RBF\widehat{X}_{t|t}\widehat{X}_{t|t}' \tag{5.17}$$

and we find the optimal F

$$F = -\left(B'RB\right)^{-1}B'RN \tag{5.18}$$

by solving the first order condition

$$\frac{\partial \Lambda^{cb}_{t|t}}{\partial F} = 0. \tag{5.19}$$

6. Households' filtering problem and expectations

As pointed out in the previous section, part of the filtering problem of the central bank consists of not knowing the expectations of households with certainty. The filtering problem of households is more straightforward. By observing the interest rate, households can infer directly what movement in the endogenous variables that are due to the actions of the central bank. The updating equation of households' filtering problem is thus given by

$$X^h_{t|t} = X^h_{t|t-1} + K^h \left[Z^h_t - W \left(AX^h_{t|t-1} + Cx^h_{t+1|t} + Bi_t \right) \right] \tag{6.1}$$

where the term in square brackets again is the innovation part of the observation. The formula for the Kalman gain K^h can be found in Appendix B.

6.1. **Inflation and output expectations.** An imperfectly informed central bank will make policy mistakes that are serially correlated, i.e. actual policy will deviate from what would be optimal if the central bank could observe the state of the economy perfectly. Since households have a different information set, they will be able to (partly) detect this and use it to forecast future policy. We model this by defining an additional state vector as the deviation of inflation and the output gap from the levels that would be attained under full information for a given exogenous state. Households can then use their estimate of the current deviation from optimal policy together with their estimate of the exogenous state X_t to forecast inflation and output tomorrow. Let

$$x_t = GX_t \tag{6.2}$$

when

$$I^h_t = I^{cb}_t = I^f_t \tag{6.3}$$

i.e. let G be the matrix that maps the exogenous state into the endogenous variables under full information. $x_t = GX_t$ then represents the optimal outcome of the endogenous variables given the constraints determined by the current state X_t. Denote the deviation from the optimal outcome η_t and let it be defined by

$$\eta_t \equiv x_t - GX_t \tag{6.4}$$

The perceived deviation from the perspective of households are then given by

$$\eta^h_{t|t} = x^h_{t|t} - GX^h_{t|t} \tag{6.5}$$

since the law of iterated expectations implies that

$$E\left[X^h_{t|t} - X_t \mid I^h_t \right] = 0 \tag{6.6}$$

The central bank's perception of (6.4) is of course zero, and any actual deviation from optimal policy must be caused by information imperfections on behalf of the central bank. Households use this information to form expectations of the next period inflation and output that are linear projections of x_{t+1} on η_t and X_t. Households expectations of inflation and output are thus given by

$$x^h_{t+1|t} = M \begin{bmatrix} X^h_{t|t} \\ \eta^h_{t|t} \end{bmatrix} \tag{6.7}$$

where

$$M = E\left[\begin{pmatrix} x_{t+1} & X_t & \eta_t \end{pmatrix}\begin{pmatrix} x_{t+1} & X_t & \eta_t \end{pmatrix}'\right] E\left[\begin{pmatrix} X_t & \eta_t \end{pmatrix}\begin{pmatrix} X_t & \eta_t \end{pmatrix}'\right]^{-1}. \tag{6.8}$$

The perceived deviation from optimal policy can be calculated by using the definition (6.5)

$$x^h_{t|t} - GX^h_{t|t} = AX^h_{t|t} + Cx^h_{t+1|t} + Bi_t - GX_{t|t} \tag{6.9}$$

$$\eta^h_{t|t} = [A + CM_1 - G]\, X_{t|t} + CM_2\eta^h_{t|t} + Bi_t \tag{6.10}$$

where M_1 and M_2 are partitions of M conformable to X_t and η_t

$$M = \begin{bmatrix} M_1 & M_2 \end{bmatrix} \tag{6.11}$$

Rearranging yields the desired expression

$$\eta^h_{t|t} = [I - CM_2]^{-1}[A + CM_1 - G]\, X_{t|t} + [I - CM_2]^{-1} Bi_t \tag{6.12}$$

7. Equilibrium dynamics, indicator accuracy and welfare

7.1. **Finding the equilibrium dynamics.** The previous two sections characterized the choice and filtering problem of the central bank and households respectively. In order to solve the model we need to find the equilibrium values of the objects $M, \Sigma_{\zeta X_{t-1}}$ and $\Sigma_{\psi\psi}$ which are determined jointly by the structural model from Section 2 and the assumed information structure. We want to find a representation of the form

$$\widetilde{X}_t = \Pi\widetilde{X}_{t-1} + \Gamma\begin{bmatrix} \mathbf{u}_t \\ \mathbf{v}_t \end{bmatrix} \tag{7.1}$$

where

$$\widetilde{X}_t = \begin{bmatrix} X_t & \widehat{X}^{cb}_{t|t} & X^h_{t|t} & Z_t & x_t & \zeta_t & \psi_t & \eta_t & i_t \end{bmatrix}' \tag{7.2}$$

Collecting the equations (2.21), (4.3), (5.3), (5.6), (5.11), (5.16), (6.1) and (6.4) gives a system of the form (7.1) where the coefficient matrices Π and Γ are partly determined by the values of $\left\{M, \Sigma_{\zeta X(t-1)}, \Sigma_{\psi\psi}\right\}$. Once we have numerical values for Π and Γ, a "new" set of $\left\{M, \Sigma_{\zeta X_{t-1}}, \Sigma_{\psi\psi}\right\}$ can be calculated from the covariance matrix of the extended state $\widetilde{X}_t$ given by the solution to the discrete Lyaponov equation

$$\Sigma_{\widetilde{x}\widetilde{x}} = \Gamma\begin{bmatrix} \Sigma_{uu} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix}\Gamma' + \Pi'\Sigma_{\widetilde{x}\widetilde{x}}\Pi \tag{7.3}$$

The coefficient matrix M can then be computed as

$$M = \Sigma_{x_{t+1}X\eta}\Sigma^{-1}_{X\eta X\eta} \tag{7.4}$$

where

$$\Sigma_{X\eta X\eta} \equiv E\left[\begin{pmatrix} X_t & \eta_t \end{pmatrix}\begin{pmatrix} X_t & \eta_t \end{pmatrix}'\right] \tag{7.5}$$

$$\Sigma_{x_{t+1}X\eta} \equiv E\left[\begin{pmatrix} x_{t+1} & X_t & \eta_t \end{pmatrix}\begin{pmatrix} x_{t+1} & X_t & \eta_t \end{pmatrix}'\right] \tag{7.6}$$

are given by the appropriate elements of $\Sigma_{\widetilde{x}\widetilde{x}}$ and $\Pi\Sigma_{\widetilde{x}\widetilde{x}}$ respectively.[4] Define the mappings S and T as

$$S \quad : \quad S(\Pi, \Gamma) = \left\{M, \Sigma_{\zeta X_{t-1}}, \Sigma_{\psi\psi}\right\} \tag{7.7}$$

$$T \quad : \quad T\left(M, \Sigma_{\zeta X_{t-1}}, \Sigma_{\psi\psi}\right) = \{\Pi, \Gamma\} \tag{7.8}$$

i.e. the mapping S computes M, $\Sigma_{\zeta X}$ and $\Sigma_{\psi\psi}$ using (7.4) and (7.3) and the mapping T computes Π and Γ using (2.21), (4.3), (5.3), (5.6),(5.16), (6.1) and (6.4). A solution to the model is a fixed point on the combined mapping

$$\left\{M, \Sigma_{\zeta X_{t-1}}, \Sigma_{\psi\psi}\right\} = S(T(M, \Sigma_{\zeta X_{t-1}}, \Sigma_{\psi\psi})) \tag{7.9}$$

which can be found numerically by iterating on (7.7) and (7.8) starting from an initial guess of $\left\{M, \Sigma_{\zeta X(t-1)}, \Sigma_{\psi\psi}\right\}$. In the terminology of Sargent (1991) the fixed point is the 'limited information rational expectations equilibrium' where the laws of motion of the process fitted by the agents coincide with the laws of motion of the actual system.[5] When the equilibrium is found we can simulate the model using (7.1). In Figure 3 below the impulse responses to a cost push shock is plotted.

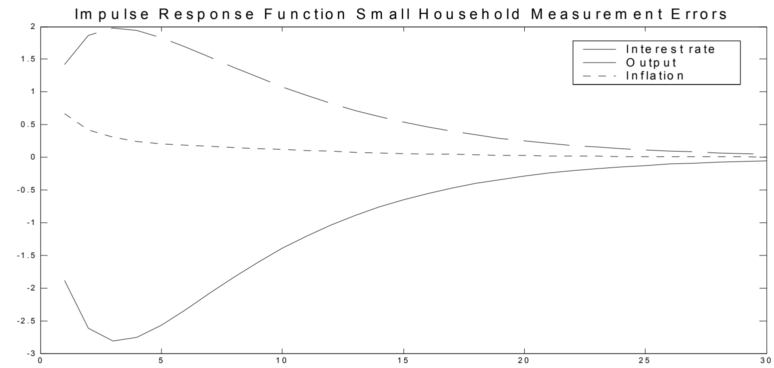

Figure 3

Inflation (dotted line) responds positively and peaks immediately, while output (solid line, which also equals the output gap) responds negatively with a slightly delayed maximum response. The delay in the maximum response of output mirrors the delayed peak of the interest rate (dashed line). The delayed maximum response is due to the measurement errors that prevents the central bank from immediately identifying the magnitude of the shock. For an eyeball inspection of whether the results of Section 3 carry over to the more realistic model we also plot the impulse responses to the same cost push shock but with less precise indicators available to households.

[4]See Ljungqvist and Sargent (2004).

[5]The equivalence between the percieved and the actual laws of motion is implied by the fixed point property of the solution but applies only to the projection coefficients, not the functional forms. It is thus possible that the equilibrium is a "reduced order equilibrium" where agents could make better estimates/forecasts if they fitted different functional forms.

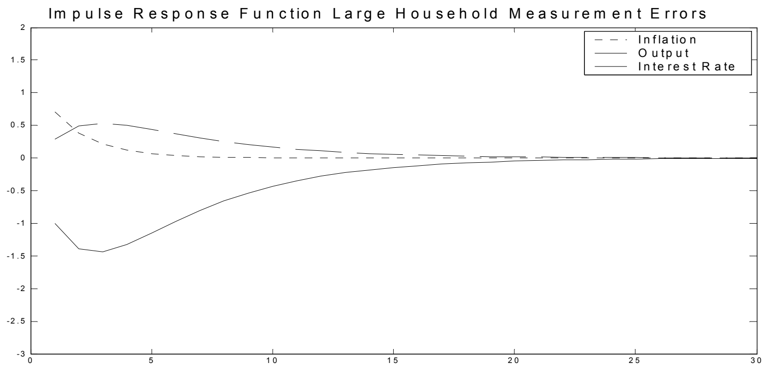

Figure 4

As Figure 4 shows, increasing the variance of the measurement errors in households' indicators do indeed induce weaker response of output and inflation, with the difference more pronounced for the response of the output gap. (The parameters used in figures are reported in the Appendix.) The next section investigates the welfare implications of changing the accuracy of households' as well as the central bank's indicators more formally.

7.2. **Computing expected losses.** With a process in the form of (7.1) it is straightforward to calculate the expected value of the loss function (2.27). The procedure is a slightly modified version of the one described in Söderlind (1999). Start by rewriting the loss function (2.27) in matrix form as

$$\Lambda_t = E_t \left[\sum_{i=0}^{\infty} \beta^i \widetilde{X}'_{t+i} \widetilde{Q}' R \widetilde{Q} \widetilde{X}_{t+i} \right]. \tag{7.10}$$

where

$$\widetilde{Q} = \left[\begin{array}{cccccc} 0 & 0 & 0 & \mathbf{0}_{2x12} & I_2 & \mathbf{0}_{2x7} \\ 0 & -\frac{\gamma}{(\gamma+\varphi)} & -\frac{(1+\varphi)}{(\gamma+\varphi)} & & & \end{array} \right] \tag{7.11}$$

The loss function can now be calculated as

$$\Lambda_t = \widetilde{X}'_t V \widetilde{X}_t + \frac{\beta}{1-\beta} trace \left(V \Gamma \left[\begin{array}{cc} \Sigma_{uu} & 0 \\ 0 & \Sigma_{vv} \end{array} \right] \Gamma' \right) \tag{7.12}$$

where V can be found by iterating on

$$V_s = \widetilde{Q}' R \widetilde{Q} + \beta \Pi' V_{s+1} \Pi. \tag{7.13}$$

We can now compute how expected losses change when we vary the accuracy of the central bank's and households' private indicators, as well as the accuracy of the indicator observable to both.

7.3. **Losses and the precision of households' indicators.** Figure 5 below confirms that what was suggested in the analysis of Section 3 also holds in the more complex diverse information setting: Increasing the noise in the private indicators of households does indeed decrease expected losses.

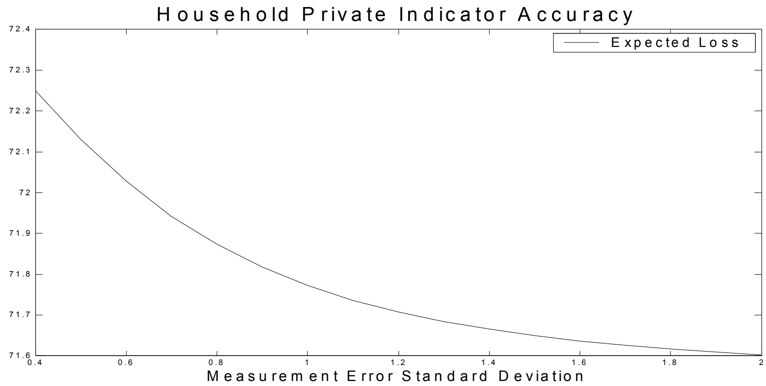

Figure 5

We experimented with varying degrees of shock persistence and accuracy of the central bank's information and the decrease in losses from an increase in noise was robust across all parameterizations.

Even when the mechanics of how less accurate household indicators leads to less volatile output gaps and inflation are clear, one may legitimately ask the question 'What prevents the agents from achieving this outcome under full information?' To answer this question, it is useful to review the argument put forward in Woodford (2001) about the welfare implications of inflation stabilization when prices are staggered. Remember the CES aggregator of goods

$$C_t = \left(\int_0^1 C_t\,(j)^{\frac{\epsilon-1}{\epsilon}}\, dj \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{7.14}$$

and note that the differentiated goods enter the index symmetrically and with decreasing marginal weight. The log-linear optimal demand schedule for each good

$$c_t(j) = -\epsilon(p_t(j) - p_t) + c_t, \tag{7.15}$$

shows that if prices were the same for all goods, it would be optimal to consume the same amount of each good. From a social perspective, the 'price' of the good is the amount of resources spent on producing it. When the individual firms' technology is symmetric and displays non-increasing returns to scale, it will also be optimal for the aggregate economy to produce the same amount of each good. In a decentralized equilibrium this will be achieved when the price for all goods are the same. In a model with staggered prices, i.e. a model where firms change prices infrequently and in a non-synchronized fashion, inflation implies changes in the relative prices of goods and $p_t(j) = p_t \; \forall \; j$ only in the steady state. Inflation thus leads to an inefficient composition of production and consumption. In a similar fashion one can argue that changes in the output gap corresponds to non-efficient variations in the relative price between goods today and goods tomorrow. An intriguing feature of this type of models is thus that the first best outcome can be achieved either through perfect price flexibility or perfect price *rigidity,* given that a benevolent and perfectly informed central bank sets the interest rate such that the economy always operate at the efficient potential output level.

So what prevents the agents from achieving the superior outcome under full information? The pricing behavior of the structural model assumes that each firm maximizes its own profit. The less desirable outcome stems from a failure of the individual firm to internalize the effect changing prices have on the composition of the consumption bundle. Less accurate information thus works as a coordinating mechanism, that is detrimental to the individual firm, but beneficial to society as a whole, by inducing the firms that do adjust prices in one period to adjust them less aggressively. This can be contrasted to the results of Bomfim (2001), who finds that even though less accurate information leads to less volatile cycles also in a standard RBC framework, they do not represent welfare improvements. In his model, the perfect information responses are optimal responses to the shocks hitting the economy.

7.4. **Losses and the precision of the central bank's indicators.** As pointed out by Pearlman (1992) the effect on expected losses of increasing the noise in the indicators available to policy makers is a priori ambiguous. The reason is that while less precise information prevents the central bank from optimally trading off inflation and output deviations, it also makes policy more inertial. To the degree that the more inertial character of policy is anticipated by the public, less precise information can improve welfare by decreasing the so-called stabilization bias of discretionary policy. The stabilization bias arises because of discretionary policy's inability to spread the adjustment to shocks over time. In Figure 6 we have plotted expected losses (on the vertical axis) when we vary the accuracy of the central bank's private indicator (along the horizontal axis).

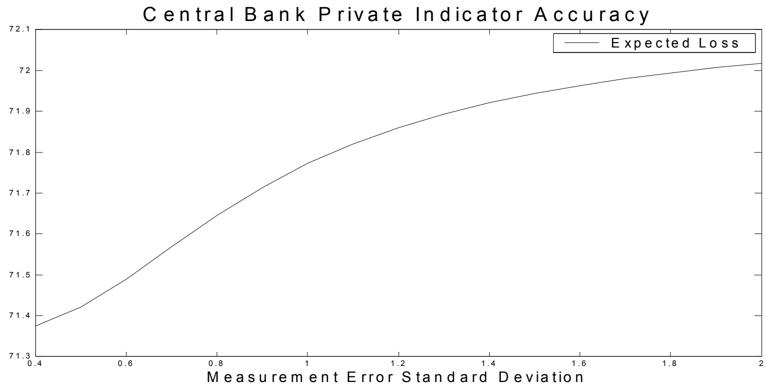

Figure 6

Figure 6 shows that losses are increasing in the magnitude of the measurement errors of the central bank's private indicator and the effect of less precise policy thus dominates. Where the graph flattens out, the magnitude of noise has become so large in the private indicator that the central bank relies entirely on the commonly observable measures of output and inflation to set policy.

7.5. **Losses and the precision of the common indicators.** Section 3 demonstrated how the positive effects of noise worked by dampening the variance of households' estimate of the cost push shock and thus dampening the variance of households' expectations. When varying the precision of the common indicators, the positive effects of noise are thus more likely to dominate when shocks are persistent and households expectations are strongly dependent on their estimate of the current state. That the persistence of shocks can reverse the direction of welfare changes when the common indicator precision is varied is illustrated by Figure 7 and 8, where we have negative welfare effects of noise with low shock persistence and positive welfare effects with high shock persistence.

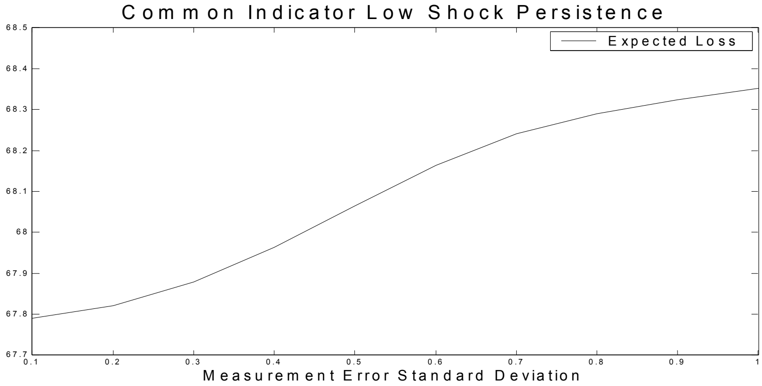

Figure 7

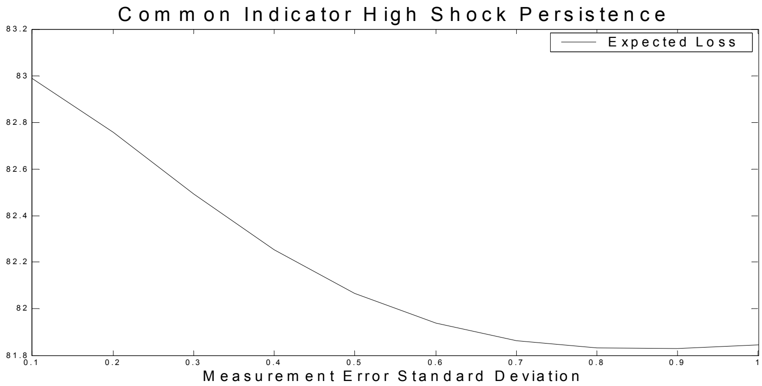

Figure 8

However, this is not a general result since the relative precision of the common indicator to the private indicators also matters. For instance, when households have very accurate private information, the precision of the common indicator becomes irrelevant to households' decisions. If at the same time the central bank is less well informed, the common indicator may be important for the precision of policy and the negative effects of noise in the common indicator then dominates. We can thus get negative effects of more noise in the common indicator for any degree of persistence, by setting the private household indicator to be accurate enough. The reverse result, but with positive effects of noise in the common indicator for (almost) all degrees of persistence, can be obtained by having a very accurate private central bank indicator.

8. Concluding remarks

In the analysis above, we have argued that assuming a common information set shared by the public and the central bank may be inappropriate when one is concerned with the welfare effects of information it self. In fact, that assumption may lead to the conclusion that monetary policy do not benefit from accurate real time data. We set up a model of diverse information to show that increasing the precision of information available to households has the opposite welfare effects of increasing the precision of information available to the central bank. We argued that less informed households decreases the relative price distortions that arises from inflation in the presence of staggered prices. The welfare effects of changing the precision of the central bank's information set are a priori ambiguous. The reason is that while less precise information prevents the central bank from optimally trading off inflation and output deviations, it also makes policy more inertial. To the degree that the more inertial character of policy is anticipated by the public, less precise information can improve welfare by decreasing the so-called stabilization bias of discretionary policy. Numerical simulations suggest that the positive effect of more precise policy dominates and that monetary policy benefits from accurate real time data. We also cannot say a priori whether the positive or negative effects will dominate when we change the precision of the indicators available to both households and the central bank, but our analysis pointed out that the positive effects of more noise are likely to be smaller, either when the households do not care much about the future, or when today's state has little predictive power over future states, i.e. when shock persistence is low. Indeed, we could produce opposite welfare effects of increased accuracy of the common information set by varying the persistence of cost push shocks, holding all other parameters fixed.

In the present paper we restricted our analysis to optimal policy under discretion. However, the results derived here should be robust to the existence of a commitment technology, though it is not obvious how one would define (or verify) commitment under diverse and imperfect information sets. The weaker responses to shocks from less informed households should be unaffected by commitment and still present the central bank with a more favorable trade-off. The theoretical possibility of positive effects from a less informed central bank would also disappear, since a commitment technology would eliminate the stabilization bias. The main result that information is good for the central bank but socially bad for households should thus be even more clear-cut if the central bank were able to commit to a future path of policy.

An implication of the negative welfare consequences of a well informed public is that the central bank should be restrictive with publishing their real time data. As an illustration, take the analysis in Coenen, Levin and Wieland (2002) who find that money can be useful as an indicator for estimating actual output, but (under the assumption of common information) the welfare gains are quantitatively small. The present paper suggests that the welfare gains could be larger if the monetary aggregates were not published, but known only to the central bank. The real time data on monetary aggregates would then increase the precision of policy without adversely affecting the trade off between inflation and output gap stabilization.

There are of course other reasons why transparency may be desirable that are not covered in the present paper. One often cited such reason is accountability. Access to the real time data that past decisions were based on is vital to a fair judgement of those decisions. In practice there should thus exist a trade off between the negative effects of transparency put forward in this paper, and the need to hold decision makers accountable.

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Appendix A. The Optimal Reset Price

Firm j resetting its price in period t maximize the expected profit function

$$E_t(j)\sum_{i=0}^{\infty}(\theta\beta)^i\left[\frac{P_t(j)}{P_{t+i}}Y_{t+i}(j)-MC_{t+i}(j)Y_{t+i}(j)\right] \tag{A.1}$$

subject to the demand constraint

$$Y_{t+i}(j) = \left(\frac{P_t(j)}{P_{t+i}}\right)^{-\epsilon} Y_{t+i} \tag{A.2}$$

where

$$Y_t = \left(\int_0^1 Y_t\left(j\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{A.3}$$

and

$$P_t = \left(\int_0^1 P_t\left(j\right)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}. \tag{A.4}$$

Substituting (A.2) into (A.1) and taking derivatives w.r.t. $P_t(j)$ gives the first order condition

$$E_t(j)\sum_{i=0}^{\infty}(\theta\beta)^i Y_{t+i}\left[\frac{1-\epsilon}{P_{t+i}}\left[\frac{P_t^*(j)}{P_{t+i}}\right]^{-\epsilon} - MC_{t+i}(j)\frac{\epsilon}{P_{t+i}}\left[\frac{P_t^*(j)}{P_{t+i}}\right]^{-\epsilon-1}\right] = 0 \tag{A.5}$$

Rearranging and simplifying yields

$$P_t^*(j)E_t(j)\left[\sum_{i=0}^{\infty}(\theta\beta)^i Y_{t+i}P_{t+i}^{\epsilon-1}\right] = \left(1+\mu\right)E_t(j)\left[\sum_{i=0}^{\infty}(\theta\beta)^i MC_{t+i}(j)P_{t+i}Y_{t+i}P_{t+i}^{\epsilon-1}\right] \tag{A.6}$$

where

$$(1+\mu) = \frac{\epsilon}{\epsilon - 1}.$$

Log linearize

$$\begin{aligned} &\left[\sum_{i=0}^{\infty}(\theta\beta)^i\right](p_t^*(j) - p_t) + \sum_{i=0}^{\infty}(\theta\beta)^i\left[y_{t+i} + (\epsilon-1)p_{t+i}\right] \\ = &\sum_{i=0}^{\infty}(\theta\beta)^i\left[p_{t+i} + mc_{t+i} + y_{t+i} + (\epsilon-1)p_{t+i}\right] \end{aligned} \tag{A.7}$$

and simplify

$$p_t^*(j) = \left(1-\beta\theta\right)E_t(j)\sum_{i=0}^{\infty}\left(\beta\theta\right)^i\left(p_{t+i} + mc_{t+i}(j)\right) \tag{A.8}$$

Appendix B. The Variance-Covariance of Cost-push Shock and Estimate

We want to find the covariance

$$\Sigma_{\varepsilon_t\varepsilon_t^h} \equiv E\begin{bmatrix}\varepsilon_t \\ \varepsilon_{t|t}^h\end{bmatrix}\begin{bmatrix}\varepsilon_t \\ \varepsilon_{t|t}^h\end{bmatrix}' \tag{B.1}$$

of the vector $\begin{bmatrix}\varepsilon_t & \varepsilon_{t|t}^h\end{bmatrix}'$ that follows the AR(1) process

$$\begin{bmatrix}\varepsilon_t \\ \varepsilon_{t|t}^h\end{bmatrix} = \begin{bmatrix}\rho & 0 \\ k & (1-k)\rho\end{bmatrix}\begin{bmatrix}\varepsilon_{t-1} \\ \varepsilon_{t-1|t-1}^h\end{bmatrix} + \begin{bmatrix}1 & 0 \\ k & k\end{bmatrix}\begin{bmatrix}u_{\varepsilon t} \\ v_t\end{bmatrix} \tag{B.2}$$

The covariance matrix is given by the solution to the Lyaponov equation[6]

$$\begin{aligned}\Sigma_{\varepsilon_t \varepsilon_t^h} &= \begin{bmatrix} 1 & 0 \\ k & k \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \begin{bmatrix} 1 & k \\ 0 & k \end{bmatrix} + \\ & \begin{bmatrix} \rho & 0 \\ k & (1-k)\rho \end{bmatrix} \Sigma_{\varepsilon_t \varepsilon_{t|t}} \begin{bmatrix} \rho & k \\ 0 & (1-k)\rho \end{bmatrix}\end{aligned} \tag{B.3}$$

Solving the resulting system of equations yields

$$\Sigma_{\varepsilon_t \varepsilon_t^h} = \tag{B.4}$$
$$\begin{bmatrix} \frac{\sigma_\varepsilon^2}{1-\rho^2} & \frac{k\sigma_\varepsilon^2}{(1-\rho^2+k\rho^2)} + \frac{\sigma_\varepsilon^2 kp}{(1-\rho^2)(1-\rho^2+k\rho^2)} \\ \frac{k\sigma_\varepsilon^2}{(1-\rho^2+k\rho^2)} + \frac{\sigma_\varepsilon^2 kp}{(1-\rho^2)(1-\rho^2+k\rho^2)} & \frac{k^2\left(\frac{\sigma_\varepsilon^2}{1-\rho_\varepsilon^2}+\sigma_v^2+\sigma_e^2-\frac{(2\rho-2k\rho)\sigma_\varepsilon^2}{(1-\rho^2+k\rho^2)}+\frac{(2\rho-2k\rho)\sigma_\varepsilon^2 p}{(1-\rho^2)(1-\rho^2+k\rho^2)}\right)}{(1-\rho^2+2k\rho^2-k^2\rho^2)} \end{bmatrix}.$$

To complete the link between noise in the households observation and the variance of inflation we need to determine the signs of the derivative of the covariance matrix (B.4) w.r.t. the variance of the measurement errors. One strategy would be to first differentiate the expression for the Kalman gain k (3.11) and the Riccati equation (3.12) and use the resulting expression to find the analytical derivatives of (B.4) w.r.t. the variance of the measurement errors. While this strategy certainly is feasible, the results are neither elegant nor intuitive and the interested reader is advised to instead use numerical methods to confirm that both the variance of estimates of the cost-push shock as well as the covariance between the estimate and the actual shock is decreasing in the variance of the measurement errors.

Appendix C. Kalman Filter Formulas

For a general reference to the Kalman filter and its properties, see Harvey (1989).

C.1. **The central bank.**

The central bank's estimate of the state vector

$$\widehat{X}_t \equiv [X_t \quad \zeta_t]' \tag{C.1}$$

is governed by the updating equation

$$\widehat{X}_{t|t} = \widehat{X}_{t|t-1} + K^{cb}\left[Z_t^{cb} - W\left(\widehat{L}\widehat{X}_{t|t-1} + Bi_t\right)\right] \tag{C.2}$$

where

$$K^{cb} = P^{cb}\widehat{L}'(\widehat{L}P^{cb}\widehat{L}' + \Sigma_{vv}^{cb})^{-1} \tag{C.3}$$

$$P^{cb} = \widehat{\boldsymbol{\rho}}(P^{cb} - P^{cb}\widehat{L}'(\widehat{L}P^{cb}\widehat{L}' + \Sigma_{vv}^{cb})^{-1}\widehat{L}P^{cb})\widehat{\boldsymbol{\rho}}' + \widehat{\Sigma}_{\mathrm{u}u} \tag{C.4}$$

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad \widehat{L} = \begin{bmatrix} A & I_2 \end{bmatrix} \tag{C.5}$$

$$\widehat{\boldsymbol{\rho}} = \begin{bmatrix} \boldsymbol{\rho} & 0 \\ \Sigma_{\zeta X_{t-1}}^{cb}\Sigma_{XX}^{cb-1} & 0 \end{bmatrix}, \widehat{\Sigma}_{\mathrm{u}u} = \begin{bmatrix} \Sigma_{\mathrm{u}u} & 0 \\ 0 & \Sigma_{\psi\psi} \end{bmatrix} \tag{C.6}$$

Using that

$$\widehat{X}_{t|t-1} = \widehat{\boldsymbol{\rho}}\widehat{X}_{t-1|t-1} \tag{C.7}$$

$$i_t = F\widehat{X}_{t|t} \tag{C.8}$$

[6]See Ljungquist and Sargent (2004).

and rearranging makes the updating operational

$$\widehat{X}_{t|t} = \left[I + K^{cb}WBF\right]^{-1}\left[I - K^{cb}W\widehat{L}\right]\boldsymbol{\widehat{\rho}}\widehat{X}_{t-1|t-1} + \left[I + K^{cb}WBF\right]^{-1}K^{cb}Z_t^{cb} \tag{C.9}$$

C.2. **Households.** Households' estimate of the state

$$X_t \equiv [\varepsilon_t \ g_t \ a_t]'$$

is governed by the updating equation

$$X^h_{t|t} = X^h_{t|t-1} + K^h\left[Z^h_t - W\left(AX^h_{t|t-1} + Cx^h_{t+1|t} + Bi_t\right)\right] \tag{C.10}$$

where

$$\begin{aligned} K^h &= P^hL(LP^hL' + \Sigma^h_{vv})^{-1} && \text{(C.11)} \\ P^h &= \rho(P^h - P^hL'(LP^hL' + \Sigma^h_{vv})^{-1}LP^h)\rho' + \Sigma_{uu}; && \text{(C.12)} \end{aligned}$$

Replace household expectations by the projection (6.7)

$$X^h_{t|t} = \rho X^h_{t-1|t-1} + K^h\left[Z^h_t - W\left(A\rho X^h_{t-1|t-1} + CM_1X^h_{t|t} + CM_2\eta^h_{t|t} + Bi_t\right)\right]$$

Substituting in the expression for $\eta^h_{t|t}$ (6.12) and rearranging makes the updating equation of households operational

$$\begin{aligned} X^h_{t|t} &= \rho X^h_{t-1|t-1} + K^h\left(Z^h_t - WA\rho X^h_{t-1|t-1}\right) && \text{(C.13)} \\ &\quad -K^hW\left(CM_1 + CM_2\left[I - CM_2\right]^{-1}\left[A + CM_1 - G\right]\right)X^h_{t|t} && \text{(C.14)} \\ &\quad -K^hW\left(CM_2\left[I - CM_2\right]^{-1}B + B\right)i_t && \text{(C.15)} \end{aligned}$$

or

$$\begin{aligned} X^h_{t|t} &= \Theta^{-1}\left(I - K^hWA\right)\rho X^h_{t-1|t-1} + \Theta^{-1}K^hZ^h_t && \text{(C.16)} \\ &\quad -\Theta^{-1}K^hW\left(CM_2\left[I - CM_2\right]^{-1}B + B\right)i_t && \text{(C.17)} \end{aligned}$$

$$\Theta \equiv \left[I + K^hW\left(CM_1 + CM_2\left[I - CM_2\right]^{-1}\left[A + CM_1 - G\right]\right)\right]$$

Appendix D. The system in AR(1) form

Collecting the equations (2.21), (4.3), (5.3), (5.6), (5.16),(5.11), (6.1) and (6.4) yields a system of the form

$$\widetilde{X}_t = \Pi\widetilde{X}_{t-1} + \Gamma\begin{bmatrix}\mathbf{u}_t \\ \mathbf{v}_t\end{bmatrix} \tag{D.1}$$

where

$$\widetilde{X}_t = \begin{bmatrix} X_t & \widehat{X}^{cb}_{t|t} & X^h_{t|t} & Z_t & x_t & \zeta_t & \psi_t & \eta_t & i_t \end{bmatrix}' \tag{D.2}$$

The coefficient matrices are given by

$$\Pi = \Pi_0^{-1}\Pi_1, \qquad \Gamma = \Pi_0^{-1}\Gamma_1 \tag{D.3}$$

$$\Pi_0 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & \Psi_1 D^{cb} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -\Theta^{-1}K^h D^h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -D & 0 & 0 & 0 & 0 \\ -A & 0 & \Psi_2 & 0 & I & 0 & 0 & 0 & \Psi_3 \\ A & 0 & 0 & 0 & -I & I & 0 & 0 & B \\ 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 \\ G & 0 & 0 & 0 & -I & 0 & 0 & I & 0 \\ 0 & -F & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{D.4}$$

$$\Psi_1 = -\left[I + K^{cb}WBF\right]^{-1} K^{cb} \tag{D.5}$$

$$\Psi_2 = -CM_2\left[I - CM_2\right]^{-1}\left[A + CM_1 - G\right] \tag{D.6}$$

$$\Psi_3 = \left(-B - CM_2\left[I - CM_2\right]^{-1} B\right) \tag{D.7}$$

$$\Pi_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Psi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Psi_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Sigma^{cb}_{\zeta X(t-1)}\Sigma^{cb-1}_{XX} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{D.8}$$

$$\Psi_4 = \left[I + K^{cb}WBF\right]^{-1}\left[I - K^{cb}W\widehat{L}\right]\widehat{\boldsymbol{\rho}} \tag{D.9}$$

$$\Psi_5 = \Theta^{-1}\left(I - K^h WA\right)\rho \tag{D.10}$$

Appendix E. Analytical solution under full information

Substitute the first order condition (2.28) into the Phillips curve (2.20) to get

$$\pi_t = \frac{(\delta\phi)^2}{\lambda}\pi_t + \beta E_t \pi_{t+1} + \varepsilon_t \tag{E.1}$$

Rearranging and substituting forward gives the solution of inflation

$$\pi_t = (1 + \delta\phi\epsilon)^{-1}\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\delta\phi\epsilon}\rho_\varepsilon\right)^k \varepsilon_t \tag{E.2}$$

Using the first order condition (2.28) again and the expression for potential output (2.14)

$$y_t - \frac{(1+\varphi)}{(\gamma+\varphi)}a_t - \frac{\gamma}{(\gamma+\varphi)}g_t = -\lambda\pi_t \tag{E.3}$$

The solution to the model under full information as a function of the exogenous state X_t is thus given by

$$x_t = GX_t \tag{E.4}$$

where

$$G=\begin{bmatrix} \frac{1}{(1+\delta\phi\epsilon)}\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\delta\phi\epsilon}\rho_{\varepsilon}\right)^{k} & 0 & 0 \\ -\frac{\epsilon}{(1+\delta\phi\epsilon)}\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\delta\phi\epsilon}\rho_{\varepsilon}\right)^{k} & \frac{\gamma}{(\gamma+\varphi)} & \frac{(1+\varphi)}{(\gamma+\varphi)} \end{bmatrix} \tag{E.5}$$

Appendix F. Parameter values used in figures

Figure	ρ_ε	ρ_g	ρ_a	$\sigma_{v\pi}$	σ_{vy1}	σ_{vy2}	σ_{vy3}	σ_ε	σ_g	σ_a
1	.8				10/25			1		
2	.8				0/25			1		
3	.8	.8	.8	2	.1	10	10	1	1	1
4	.8	.8	.8	2	.1	10	.5	1	1	1
5	.5	.5	.5	1	-	1	1	1	1	1
6	.5	.5	.5	1	1	1	-	1	1	1
7	.1	.1	.1	1	1	-	1	1	1	1
8	.5	.5	.5	1	1	-	1	1	1	1

Parameters with common values in all figures

β	γ	φ	ϵ	θ
.99	2	$1\frac{1}{2}$	10	$\frac{3}{4}$