Modelling Information, Learning and Expectations in Macroeconomics

Lecture 11

New York University

Spring 2008

Information, Learning and Expectations in Macroeconomics

Today:

Monetary policy makers and models:
Learning the slope of the Phillips Curve, Sargent (1999)

- One model, different parameter estimates
- Produces episodes that looks like the 1970s and the 1980s.

Recursive model validation with competing models, Cho and Kasa (2007)

- Different models, one well fitting but complex, one worse fitting but simple
- Preference for simple model

The Conquest of American Inflation, Sargent (1999)

What explains the increase and then decrease of inflation in the period 1960-1990?
Two competing explanations:

- Policy makers first thought that there was an exploitable Phillips curve trade off, but learned from Lucas and others that that this was not the case and found a way to commit themselves to low inflation in the 80's
- Macro econometric policy evaluation never left the (Fed) building: It only happened to be that at the moment of the Volcker disinflation, the optimal inflation-unemployment trade-off implied low inflation.

A Model of (lack of) commitment

Private agents' pay-off function

$$u(\xi, x, y) = -.5\left[(y-\xi)^2 + y^2\right]$$

Governments period pay-off function

$$-.5\left(U^2 + y^2\right) \tag{1}$$

Expectations augmented Phillips Curve

$$U = U^* - \theta(y - x), \quad \theta > 0 \tag{2}$$

Substituting (1) into (2) gives

$$r(x, y) = -.5\left[(U^* - \theta(y-x))^2 + y^2\right]$$

A model of (lack of) commitment: Equilibrium

Rational Expectations Equilibrium: (2) and $y = x$
Government's best response

$$y = \frac{\theta}{\theta^2 + 1} U^* + \frac{\theta^2}{\theta^2 + 1} x$$

taking x as given

- Nash outcome: $y^N = x^N = \theta U, \;\; U = U^*$
  - $r\left(y^N, x^N\right) = -.5\left(1 + \theta^2\right) U^{*2}$
- Ramsey outcome $(max_y r(y, y)) : \; y^R = x^R = 0, \;\; U = U^*$
  - $r\left(y^R, x^R\right) = -.5 U^{*2}$

Learning to play Nash

Let agents form inflation expectations using RLS learning

$$x_t = x_{t-1} + (t-1)^{-1} (y_{t-1} - x_{t-1})$$

represent policy makers best response as $B(x_t)$ and substitute into RLS updating equation

$$x_t = x_{t-1} + (t-1)^{-1} (B(x_t) - x_{t-1})$$

If

$$\frac{dB(x_t)}{dx_t} < 1$$

the differential equation

$$\frac{dx}{dt} = B(x_t) - x_t$$

will converge to $x_t = B(x_t)$.

A Dynamic Policy Problem

The "Original Phelps Problem":

$$
\begin{aligned}
V^{g}(U, y) &= (1-\delta) \sum_{t=0}^{\infty} \delta^{i-1} \left[-.5\left(U_t^2 + y_t^2\right)\right] \\
U &= U^{*} - \theta\left(y_t - x_t\right)
\end{aligned}
$$

Original adaptive expectations

$$x_t - x_{t-1} = (1-\lambda)\left(y_t - x_{t-1}\right)$$

If $\delta = 0, \lim_{t\to\infty} y_t = 0$ (Ramsey outcome)

A Dynamic Policy Problem

The "General Phelps Problem":

$$y_t = h(\gamma_t) X_{t-1} + v_t$$

Reduced form (Classical direction of fit) Phillips curve

$$U_t = (\alpha +) \ldots \begin{bmatrix} \gamma_1 & \gamma'_{-1} \end{bmatrix} \begin{bmatrix} y_t \\ X_{t-1} \end{bmatrix} + \varepsilon_{Ct}$$

Keynesian direction of fit

$$y_t = \begin{bmatrix} \beta_1 & \beta'_{-1} \end{bmatrix} \begin{bmatrix} U_t \\ X_{t-1} \end{bmatrix} + \varepsilon_{Kt}$$

X_{t-1} contains lags of U_t and y_t

A Dynamic Policy Problem

The policy makers update their estimates of the reduced form Phillips curves according to

$$\begin{aligned} \gamma_t &= \gamma_{t-1} + g_t R_{XC,t} X_{Ct} \left(U_t - \gamma'_{t-1} X_{Ct} \right) \\ R_{XC,t} &= R_{XC,t-1} + g_t \left(X_{Ct} X'_{Ct} - R_{XC,t-1} \right) \end{aligned}$$

Maximizing "anticipated utility" is setting $y_t = h(\gamma_t) X_{t-1}$ where h is the optimal policy function if γ_t was the true model coefficients

$$y_t = h(\gamma_t) X_{t-1}$$

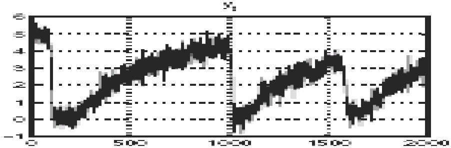
Inflation expectations given by the "Fed watcher" assumption:

$$x_t = h(\gamma_t) X_{t-1}$$

Inflation dynamics

With decreasing gain $g_t = t^{-1}$ model converges to Nash equilibria
With constant gain $g_t = g$ we get slow drift towards Nash, then sudden reversals towards Ramsey outcomes

- Constant gain will look optimal as coefficient will appear to drift (SCE)



What is going on?

Why does a misspecified model (i.,e. the belief in drift) produce better outcomes?

- Combination of parameters suggest that there is no long run trade-off
- Better outcomes relies on policy makers not knowing the true Phillips curve so there for do not try to exploit private agents' expectations

Summing up "The Conquest...." story

It is an exercise in positive economics:

- With a Keynesian direction of fit, recursive constant gain estimation of the Phillips curve can fit the 1960-1980s US data.
- Inflation will slowly increase until the trade off becomes less and less attractive and then data accumulates to suggest that there is not long run trade off and inflation falls towards the Ramsey outcome

Sargent expresses hope that the Fed is smarter today and that we will not experience a drift towards higher inflation again

Learning and Model Validation, Kasa and Cho (2007)

Agents are still econometricians, only a little better trained

- Agents use constant gain learning
- Agents also engage in model validation, drawing on methods to compare misspecified models

Cho and Kasa are not discussing specific episode of history.

"It takes a model to beat a model"

All models false, we need to be able to choose between misspecified models. Kasa and Cho suggest selecting models based on the Kullback-Leibler Information Criteria (*KLIC*)

Kullback-Leibler divergence

$$D_{KL}(P \parallel Q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

KLIC of model $q(x)$ is found by letting $p(x)$ represent the true data generating process (DGP)

- It is very useful that we can minimize the *KLIC* of a given model or compare two models using *KLIC* without knowing the true DGP

Choosing between models

Two models relative KLIC can in linear (single equation) Gaussian models be represented as

$$\left[\log\left(\sigma_2^2\right) - \log\left(\sigma_1^2\right)\right] + \frac{1}{T}\left(K_2 - K_1\right)$$

Kasa and Cho let agents use this criteria to select "reference model" model

- Switching reference model requires that KLIC difference is above a certain threshold ρ
  - $\Longrightarrow$ implicit cost of switching
- Both models always estimated
- Cost of complexity statistical, not cognitive

Model set up

Same as in conquest

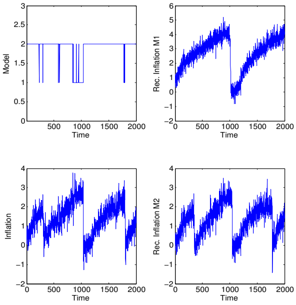
$$\begin{aligned} V^g(U,y) &= (1-\delta)\sum_{t=0}^{\infty}\delta^{i-1}\left[-.5\left(U_t^2+y_t^2\right)\right] \\ U_t &= U^* - \theta\left(y_t - x_t\right) + v_{1t} \\ y_t &= h_M(\gamma)X_{M,t-1} \\ x_t &= h_M(\gamma)X_{M,t-1} \end{aligned}$$

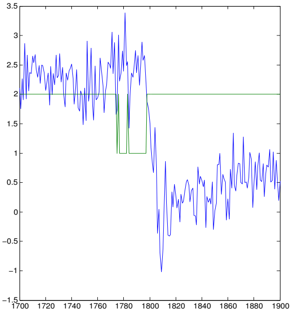
Policy makers fit two models using constant gain learning

$$\begin{aligned} M_1 &: \quad U_t = \gamma_0 + \gamma_1 y_t + \xi_{1t} \\ M_2 &: \quad U_t = \gamma_0 + \gamma_1 y_t + \sum_{l=1}^{2}\lambda_{tl} y_{t-1} + \xi_{2t} \end{aligned}$$

Example 2A: Escape Dynamics and Model Selection

$\gamma = .02 \quad \rho = 1.1$ Model 1 = Static Model 2 = Dynamic





What's going on?

- Most of the time, dynamic (complex) model in use
- When inflation approaches Nash, policy makers switches to using static model as reference model since residuals are less serially correlated (the true model has no dynamics)
- Model validation dynamics look like recursive learning dynamics of dynamic model, but escapes more frequent if no model validation and policy makers use dynamic model
- Escape dynamics less frequent because of simple model

Summing up today

Large players using constant gain learning can result in interesting and unexpected dynamics

Is the Conquest of American Inflation a good story?

- It would be nice if there was some direct evidence for the sudden switch in beliefs, perhaps from Fed minutes or similar