Modelling Information, Learning and Expectations in Macroeconomics

Lecture 5

New York University

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Modelling Information, Learning and Expectations in Macroeconomics

Today:

- Hello
- Signal extraction from endogenous variables (Svensson and Woodford, JEDC 2004)
  - Application: Is there useful information about the business cycle in the term structure of interest rates? (Nimark 2007)
- Homework

Signal extraction from endogenous variables

The usual form of the system:

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t \\ Z_t &= DX_t + \mathbf{v}_t \end{aligned}$$

But what if Z_t depends on $X_{t|t}$?

An application:

A joint model of the macro economy, the term structure and monetary policy under imperfect information

- Policy maker wants to minimize a loss function with unobservable arguments (e.g. output gap)
- The question: Is there information in the term structure that can be used for quarter-to-quarter monetary policy?

The informational set up:

- Central bank observes noisy measures of lagged output and inflation and contemporaneous bond yields
- The central bank's information set is a strict subset of the private sectors information set
  - Consumption and price setting decisions are conditional on the true state and the central bank's estimate of the true state
- Bond market reflects some information that is unknown to the central bank, but also noise.

The Macro Model

- Households consume goods and supply labour
- Habits in consumption
- Firms set prices to maximize profits in monopolistically competitive markets
- Price setting subject to the Calvo (1983) mechanism and a fraction of firms use lagged inflation rule-of-thumb

The linearised model

$$y_t = \mu_{yf} E_t y_{t+1} + \mu_{yb} y_{t-j} - \delta \left[i_t - E_t \pi_{t+1} \right] + \varepsilon_t^y$$

$$\begin{aligned} \pi_t &= \mu_{\pi f} E_t \pi_{t+1} + \mu_{\pi b} \pi_{t-1} + \kappa mc_t + \varepsilon_t^\pi \\ mc_t &= (\varphi + \gamma) y_t + \eta (1 - \gamma) y_{t-1} - (1 + \varphi) a_t \\ a_t &= \rho_a a_{t-1} + \varepsilon_t^a \end{aligned}$$

or

$$\begin{bmatrix} X_{1,t+1} \\ E_t \left[X_{2,t+1} \right] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_t$$

The Policy Maker's problem

Minimize the loss function

$$\mathbf{L}_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[\lambda_y (y_{t+k} - \overline{y}_{t+k})^2 + \pi_{t+k}^2 + \lambda_i (i_{t+k} - i_{t+k-1})^2 \right] \right]$$

by choosing F in the policy function

$$i_t = F X_{1,t|t}$$

where $X_{1,t|t}$ is the policy maker's estimate of the state

$$\begin{aligned} X_{1,t|t} &= X_{1,t|t-1} + K \left(Z_t - D_1 X_{1,t} - D_2 X_{1,t|t} \right) \\ X_{1,t} &= \left[a_t, y_{t-1}, \pi_{t-1}, \varepsilon_t^y, \varepsilon_t^\pi, i_{t-1}, \Delta i_t, \mathbf{v}_t \right]' \\ Z_t &= D_1 X_{1,t} + D_2 X_{1,t|t} = \left[y_{t-1} + v_t^y \quad \pi_{t-1} + v_t^\pi \quad \mathcal{Y}_t \right]' \end{aligned}$$

and $\mathcal{Y}_t$ is a vector of bond yields

Solving the model

Conceptually, we can split the model into three parts:

1. The households consumption and price setting problem
2. The central bank's filtering and control problem
3. Pricing bonds using no arbitrage and the stochastic discount factor implied by the households' utility function

The three parts are interdependent and need to be solved simultaneously

Consumption and price setting

First, take policy function and filter as given

$$\begin{aligned} i_t &= FX_{t|t} \\ X_{1,t|t} &= X_{1,t|t-1} + K\left(Z_t - D_1X_{1,t} - D_2X_{1,t|t}\right) \end{aligned}$$

and solve for expectations in the linearised macro model. That is, we want a solution of the form

$$\begin{aligned} X_{1,t} &= HX_{1,t-1} + JX_{1,t-1|t-1} + C\varepsilon_t \\ X_{2,t} &= G^1X_{1,t} + \left(G - G^1\right)X_{1,t|t} \end{aligned}$$

where H and J are defined as

$$\begin{aligned} H &= A_{11} + A_{12}G^1 \\ J &= B_1F + A_{12}\left(G - G^1\right) \end{aligned}$$

F and G same as in full information model

Consumption and price setting

$$\begin{bmatrix} X_{1,t+1} \\ E_t\left[X_{2,t+1}\right] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} F X_{t|t} + \begin{bmatrix} C \\ 0 \end{bmatrix} \varepsilon_t$$

Use the conjectured form

$$X_{2,t} = G^1 X_{1,t} + \left(G - G^1\right) X_{1,t|t}$$

to get

$$\begin{aligned} & \begin{bmatrix} I \\ G_1 \end{bmatrix} X_{1,t+1} + \begin{bmatrix} 0 \\ \left(G - G^1\right) \end{bmatrix} E_t X_{1,t+1|t+1} \\ = & \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I \\ G_1 \end{bmatrix} X_{1,t} + \begin{bmatrix} 0 \\ \left(G - G^1\right) \end{bmatrix} X_{1,t|t} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} F X_{t|t} \end{aligned}$$

Consumption and price setting

Using

$$\begin{aligned} X_{1,t} &= HX_{1,t-1} + JX_{1,t-1|t-1} + C\varepsilon_t \\ X_{1,t|t} &= (H+J)X_{1,t-1|t-1} + K\left(Z_t - D_1 X_{1,t} - D_2 X_{1,t|t}\right) \end{aligned}$$

and take expectations with respect to time t

$$\begin{aligned} &\begin{bmatrix} I \\ G^1 \end{bmatrix} \left[HX_{1,t} + JX_{1,t|t}\right] + \\ &\begin{bmatrix} 0 \\ \left(G - G^1\right) \end{bmatrix} KD_1 \left[HX_{1,t} + JX_{1,t|t}\right] + [H+J]\, X_{t|t} \\ = &\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I \\ G^1 \end{bmatrix} X_{1,t} + \begin{bmatrix} 0 \\ \left(G - G^1\right) \end{bmatrix} X_{1,t|t} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} FX_{t|t} \end{aligned}$$

Consumption and price setting

Collect all terms in the lower block involving $X_{1,t}$

$$G^1 H X_{1,t} + \left(G - G^1\right) K D_1 H X_{1,t} = A_{21} X_{1,t} + A_{22} G^1 X_{1,t}$$

Equate coefficients

$$\begin{aligned} G^1 H + \left(G - G^1\right) K D_1 H &= A_{21} + A_{22} G^1 \\ A_{22}^{-1} \left[G^1 H + \left(G - G^1\right) K D_1 H - A_{21}\right] &= G^1 \end{aligned}$$

We thus have an implicit equation for G^1 (for given Kalman gain K)

Consumption and price setting

Similar procedure can be used to find G, but starting from

$$X_{1,t} = X_{1,t|t}$$

and

$$X_{2,t} = GX_{1,t}$$

The Policy Maker's Filtering problem

$$X_{1,t|t} = X_{1,t|t-1} + K\left(Z_t - D_1 X_{1,t|t-1} - D_2 X_{1,t|t}\right)$$

Two ways to handel simultaneity i)

$$X_{1,t|t} = (I + KD_2)^{-1} X_{1,t|t-1} + (I + KD_2)^{-1} K\left(Z_t - D_1 X_{1,t|t-1}\right)$$

or equivalently ii)

$$X_{1,t|t} = X_{1,t|t-1} + K\left(D_1 X_{1,t} - D_1 X_{1,t|t-1}\right)$$

The steady state Kalman gain is then given by

$$\begin{aligned} K &= PD_1'\left(D_1 PD_1'\right)^{-1} \\ P &= H(P - PD_1'\left(D_1 PD_1'\right)^{-1} D_1 P)H' + CC' \end{aligned}$$

The law of motion of the state

We now have a system in the form

$$\begin{aligned} X_{1,t} &= HX_{1,t-1} + JX_{1,t-1|t-1} + C\varepsilon_t \\ X_{1,t|t} &= (I + KD_1)(H + J)X_{1,t|t-1} + K\left(D_1 X_{1,t} + \mathbf{v}_t\right) \\ X_{2,t} &= G^1 X_{1,t} + \left(G - G^1\right) X_{1,t|t} \end{aligned}$$

The $\overline{X}_t = \left[\begin{array}{cc} X_{1,t} & X_{1,t|t} \end{array}\right]'$ of the model then follows

$$\begin{aligned} \overline{X}_t &= \left[\begin{array}{cc} H & J \\ KD_1 H & [(H+J) + KD_1 J - KD_1(H+J)] \end{array}\right] \overline{X}_{t-1} \\ &\quad + \left[\begin{array}{c} C \\ KD_1 C \end{array}\right] \varepsilon_t \end{aligned}$$

But we still need to find D_1

Finding bond yields

Use the stochastic discount factor implied by the model

$$E_t M_{t+1} \equiv E_t \beta \frac{U_{ct+1} P_t}{U_{ct} P_{t+1}}$$

That the price in period t of a dollar delivered in period $t+n$ is given by $E_t M_{t+1} M_{t+2} ... M_{t+n}$.

$$\log E_t M_{t+1} = \log \beta - \gamma E_t c_{t+1} + (\gamma - \eta + \gamma\eta) c_t + \eta(1-\gamma) c_{t-1} - E_t \pi_{t+1} - \frac{\sigma_m^2}{2}$$

where we used that

$$\log E_t M_{t+1} = E_t m_{t+1} - \frac{\sigma_m^2}{2}$$

and that

$$m_{t+1} = -\gamma c_{t+1} + (\gamma - \eta + \gamma\eta) c_t + \eta(1-\gamma) c_{t-1} - \pi_{t+1}$$

In equilibrium

$$E_t m_{t+1} + i_t = 0$$

Finding bond yields

In logs the price b_t^n of a dollar delivered n periods ahead is given by

$$b_t^n = \log E_t M_{t+1} + \log E_t M_{t+2} + ... \log E_t M_{t+n} + nv_t^n$$

Using that $E_t m_{t+1} = -i_t$

$$\begin{aligned} b_t^n &= -E_t i_t - E_t i_{t+1} + ... - E_t i_{t+n-1} + k + nv_t^n \\ &= -\overline{F}\overline{X}_t - \overline{F} M \overline{X}_t + ... - \overline{F} M^{n-1} \overline{X}_t + k + nv_t^n \\ &= \overline{B}_n \overline{X}_t + k + nv_t^n \end{aligned}$$

where

$$\overline{F} = \left[\begin{array}{cc} \mathbf{0}_{1x7} & F \end{array} \right]$$

and M is the coefficient matrix from the law of motion of the state $\overline{X_t}$. The yield of an n period bond is found by multiplying the log price with $-1/n$.

Two perspectives on the bond yield equation

$$\mathcal{Y}_t = \mathbf{k} + \begin{bmatrix} -\overline{B}_1 \\ \vdots \\ -\frac{1}{n}\overline{B}_n \end{bmatrix} \overline{X}_t + \mathbf{v}_t^{\mathcal{Y}}$$

Since $\mathbf{v}_t^{\mathcal{Y}} \in \mathbf{v}_t \in X_{1,t}$ we can write bond yields as a function of state $\overline{X_t}$ so that

$$\begin{aligned} Z_t &= \begin{bmatrix} y_{t-1} + v_t^y & \pi_{t-1} + v_t^{\pi} & \mathcal{Y}_t \end{bmatrix}' \\ &= D_1 X_{1,t} + D_2 X_{1,t|t} \end{aligned}$$

Solving the model

Iterate on the algorithm:

1. The households consumption and price setting problem
   1.1 Find G, G^1 for given K, F, D_1, D_2
2. The central bank's filtering and control problem.
   2.1 Find K, F, for given G, G^1, D_1, D_2
3. Pricing bonds using no arbitrage and the stochastic discount factor implied by the households' utility function
   3.1 Find D_1, D_2 for given G, G^1, K, F

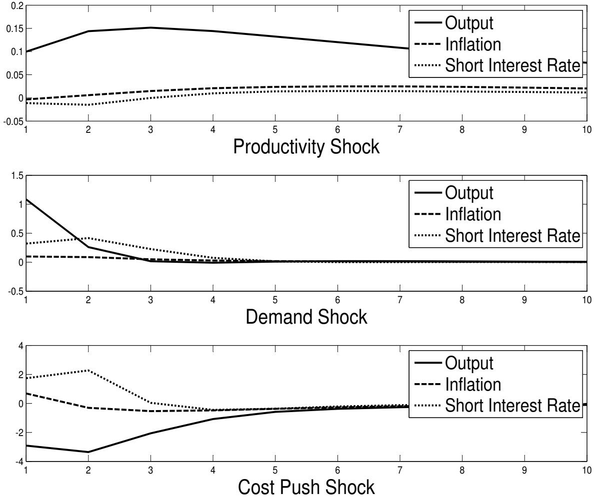

Figure: Estimated impulse response to technology, demand and cost push shocks

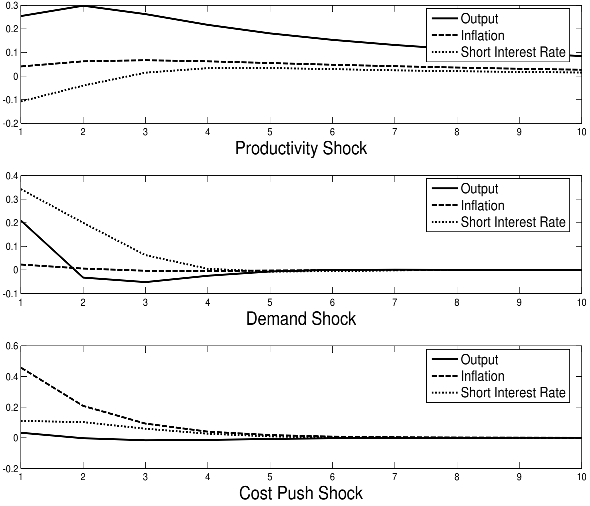

Figure: Impulse response with perfectly informed central bank

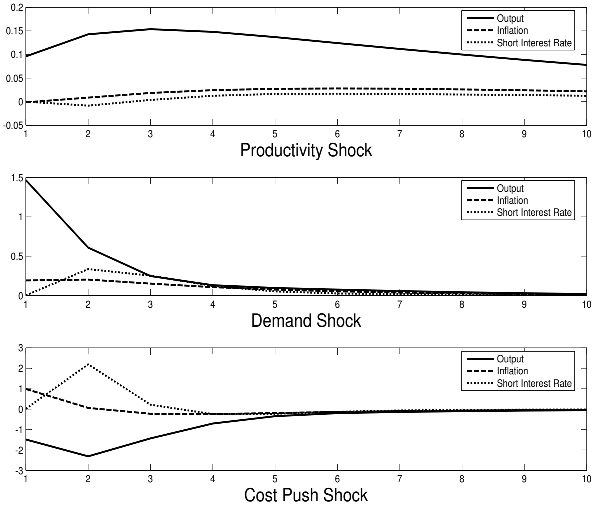

Figure: Impulse response without term structure information

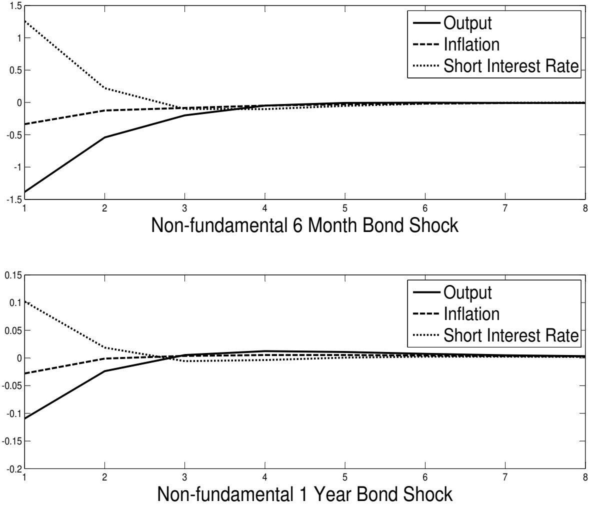
1.5
1
0.5
0
-0.5
-1
-1.5
1
2
3
4
5
6
7
8
Output
Inflation
Short Interest Rate
Non-fundamental 6 Month Bond Shock
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
1
2
3
4
5
6
7
8
Output
Inflation
Short Interest Rate
Non-fundamental 1 Year Bond Shock

Figure:

Table 2 Variance Decomposition
(2.5%-97.5% probability intervals)

	ε_t^a	ε_t^y	ε_t^π	v_t^{ycb}	$v_t^{\pi cb}$	$v_t^{\mathcal{Y}2}$	$v_t^{\mathcal{Y}4}$
y_t	0.79 (0.69–0.84)	0.12 (0.08–0.20)	0.01 (0–0.04)	0.07 (0.04–0.08)	0 (0–0)	0.01 (0.01–0.02)	0 (0–0)
π_t	0.63 (0.54–0.71)	0.05 (0.01–0.12)	0.01 (0–0.04)	0.30 (0.15–0.38)	0 (0–0)	0.01 (0–0.02)	0 (0–0)
i_t	0.17 (0.09–0.24)	0.51 (0.41–0.54)	0.05 (0–0.12)	0.11 (0.05–0.17)	0 (0–0.04)	0.17 (0.13–0.21)	0 (0–0)
$\mathcal{Y}_t^2$	0.15 (0.07–0.23)	0.38 (0.31–0.43)	0.03 (0–0.06)	0.06 (0.03–0.09)	0 (0–0.01)	0.39 (0.30–0.48)	0 (0–0)
$\mathcal{Y}_t^4$	0.03 (0.02–0.07)	0.07 (0.05–0.11)	0 (0–0.02)	0.01 (0.01–0.02)	0 (0–0)	0.01 (0.01–0.02)	0.87 (0.79–0.89)