

# Modelling Information, Learning and Expectations in Macroeconomics

## Lecture 9

New York University

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# Information, Learning and Expectations in Macroeconomics

Today:

Policy with dispersed information

Modelling philosophy: Real and informational heterogeneity

## Policy with dispersed information

*Optimal Monetary Policy with Imperfect Common Knowledge*,  
Adam JME (2007)

*Optimal Monetary Policy with Uncertain Fundamentals and  
Dispersed Information*, Lorenzoni, MIT mimeo (2008)

*Policy with Dispersed Information*, Angeletos and Pavan, mimeo  
MIT Northwestern (2007)

# Common Themes

Optimality depends on some measure of average welfare

Welfare in turn depends on level and composition effects

- ▶ Level effect: Information may cause agents to consume/invest more or less than they would under full information
- ▶ Composition effect: Dispersed information may cause inefficient relative price changes so that composition of consumption/factor input bundles are inefficient

# Adam 2007

General set-up:

- ▶ Woodford (2002)-style imperfect information flexible prices economy
  - ▶ Supply and demand shocks added
  - ▶ Endogenous information choice a la Sims Rational inattention
- ▶ Benevolent and perfectly informed central bank

## The model of Adam

Households' problem

$$\max_{Y,L} U(Y) - \nu V(L)$$

subject to

$$0 = WL - \Pi - T - PY$$

$U' > 0$ ,  $U'' < 0$ ,  $\lim_{Y \rightarrow \infty} U'(Y) = 0$ ,  $V' > 0$ ,  $V'' < 0$  and  
 $V'(0) < U'(0)$

# The model of Adam

The pricing decision

$$p(i) = E [p + \xi(y - y^*) + u \mid I^i]$$

where

$$y = q - p$$

$u$  and  $y^*$  are exogenous supply and mark up shocks respectively and  $q$  is money supply

## The model of Adam

Monetary policy

$$\max_q -E \left[ (y - y^*)^2 + \lambda p^2 \right]$$

subject to

$$p(i) = E \left[ (1 - \xi) p + \xi q - \xi y^* + u \mid I^i \right]$$

where we used that  $y = q - p$

## Rational inattention: A short cut

Firms need to process information about  $y^*$  and  $u$  in order to implement the pricing decision

$$p(i) = E [(1 - \xi) p + \xi q - \xi y^* + u \mid I^i]$$

It can be shown to be optimal to process information about a linear combination instead

$$\zeta' Z = \zeta' \begin{bmatrix} y^* \\ u \end{bmatrix}$$

so that the signal  $s(i)$  is given by

$$s(i) = \zeta' Z + \eta(i)$$

## Rational inattention: A short cut

Firms optimal expectation is given by

$$E [\zeta' Z | s(i)] = ks(i)$$

where  $k$  is the Kalman gain given by

$$k = \frac{\text{var} [\zeta' Z]}{\text{var} [\zeta' Z] + \sigma_n^2} = \left(1 - 2^{-2K}\right)$$

## Rational inattention: A short cut

The optimal price of firm  $i$   $p^*(i)$  can be written as a function of higher order expectations of  $q, y^*$  and  $u$

$$p^*(i) = \sum_{m=0}^{\infty} (1 - \xi)^m \left( \xi q^{(m)} - \xi y^{*(m)} + u^{(m)} \right)$$

We can write any higher order combination of

$$E \left[ \left( \xi q^{(m)} - \xi y^{*(m)} + u^{(m)} \right) \mid I^i \right] = k^{m+1} s(i)$$

which gives an aggregate price level

$$p = \frac{k}{1 - (1 - \xi)k} (\xi q - \xi y^* + u)$$
$$y - y^* = \frac{1 - k}{1 - (1 - \xi)k} (q - y^*) - \frac{k}{1 - (1 - \xi)k} u$$

## Optimal monetary policy

$$\max_q -E \left[ (y - y^*)^2 + \lambda p^2 \right]$$

subject to

$$p = \frac{k}{1 - (1 - \xi)k} (\xi q - \xi y^* + u)$$
$$y - y^* = \frac{1 - k}{1 - (1 - \xi)k} (q - y^*) - \frac{k}{1 - (1 - \xi)k} u$$

gives policy reaction function

$$q = au + y^*$$

where

$$a = \frac{(1 - k)k - \lambda \xi k^2}{(1 - k)^2 + \lambda \xi^2 k^2}$$

## Optimal policy

$$q = au + y^*, \quad a = \frac{(1-k)k - \lambda\xi k^2}{(1-k)^2 + \lambda\xi^2 k^2}$$

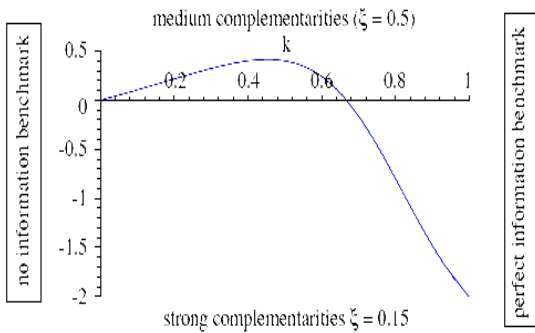
1. Pure output gap stabilisation:  $\lambda = 0$

$$a_y = \frac{k}{1-k}$$

2. Pure price level stabilisation:  $\lambda = \infty$

$$a_p = -\frac{1}{\xi}$$

For  $\lambda \notin \{0, \infty\}$   $a$  is a convex combination of  $a_y$  and  $a_p$



## Summing up Adam (2007)

Optimal policy depends on information processing capacity of price setters

A few tricks:

- ▶ Formulate multivariate information processing problem as equivalent scalar process
- ▶ Inefficient price dispersion increasing in price level variance
- ▶ Assume perfectly informed central bank for most of analysis (certainty equivalence also holds...)

# *Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information*

Lorenzoni (2008)

General set-up

- ▶ Monopolistically competitive macro economy
- ▶ Common and idiosyncratic productivity shocks
- ▶ Noise in public signal about common productivity
- ▶ No information asymmetry, i.e. central bank uses only publicly observable information

## Model overview Lorenzoni (2008)

Price setting

$$p_{it} = \kappa_p + E_{i(t,l)} [\bar{p}_{it} + \gamma c_{it} + \eta n_{it}] - a_{it}$$

Consumption

$$c_{it} = \kappa_c + E_{i(t,ll)} [c_{it+1}] - \gamma^{-1} (r_t - E_{i(t,ll)} [\bar{p}_{it+1}]) + \bar{p}_{it}$$

Information sets

$$I_{i(t,l)} = \{s_t, x_{it}, a_{t-1}\}, I_{i(t,ll)} = \{P_{jt}\}$$

$$s_t = \theta_t + e_t$$

$$x_{it} = \theta_t + \varepsilon_{it}$$

where  $\theta_t$  is the innovation in the persistent productivity process

$$a_t = \rho a_{t-1} + \theta_t$$

## The Monetary Policy Channel

How can policy effect the use of information in this model?

Policy follows a rule of the form

$$\begin{aligned}r_t &= \xi_0 + \xi_a a_{t-1} + \xi_m (m_{t-1} - \hat{m}_{t-1}) \\ \hat{m}_t &= \mu_0 + \mu_a a_{t-1} + \mu_\theta \theta_t + \mu_e + e_t\end{aligned}$$

By promising to respond in a particular way in the future to the aggregate noise  $e_t$  agents will behave as if they put less weight on public signal and more weight on their private signals

- ▶ Can achieve "full aggregate stabilisation"
  - ▶ However this is not optimal as it leads to inefficiently large price dispersion

## Optimal policy with dispersed information, Angeletos and Pavan (2007)

General framework for analyzing use of information from efficiency perspective

$$u_i = U(k_i, K, \sigma_k, \theta) - \tau_i$$

Agents receive two signals, one private and one public

$$x_i = \theta + \xi_i$$

$$y = \theta + \varepsilon$$

Decision rule is in the form

$$k_i = \kappa(x_i, y)$$

which can be decomposed into

$$k_i = \hat{k}_i + \epsilon + v_i$$

where  $\hat{k}_i$ ,  $\epsilon$  and  $v$  are responses to the fundamental, and common and idiosyncratic noise resp.

## How does taxation influence use of information?

Derive first best outcome:

$$k_i = \kappa^*(x_i, y)$$

(If full information solution implies that  $\kappa = \kappa^*$  no externalities,  $\kappa \neq \kappa^*$  implies externalities.)

Define a tax structure so that

$$\tau_i = T(k_i, K, \sigma_k, \theta)$$

The tax-man has as many instruments as there are arguments in utility function so he can control responses perfectly and choose  $T$  to replicate  $\kappa^*$

Argument rests on observability of actions.

# Modelling Information Imperfections

How do we calibrate information accuracy?

What are the free parameters..

..and are they really free?

## Expectations and Assumptions

The rational expectations revolution was partly successful because it eliminated agents expectations as a free parameter, i.e. one less thing to argue about in seminars.

Is information imperfections literature introducing new free parameters?

- ▶ Rational inattention (Sims etc): Information channel capacity
- ▶ Inattentiveness (Mankiw and Reis): The frequency of information gathering
- ▶ Imperfect and exogenous information (most of this course): The variances of the noise in the filtering problem

## Expectations and Assumptions

One approach: Let agents observe quantities and shocks in their immediate environment perfectly and make these idiosyncratic.

- ▶ Answers concerns that agents immediate environment provides a sufficient statistic for optimal decision making
- ▶ Can potentially provide a way to calibrate a "lower bound" on precision (or upper bound on noise).

# Real and Informational Heterogeneity

Graham and Wright (2007): Linearised RBC model with idiosyncratic capital

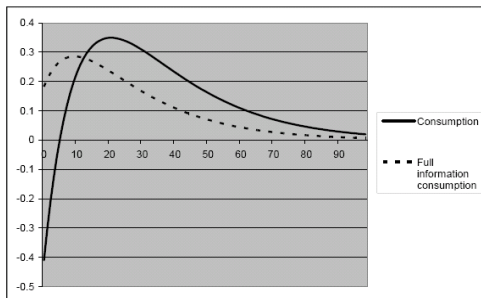
Nimark (2008): New Keynesian model with idiosyncratic marginal cost

# *Information, Heterogeneity, and Market Incompleteness, Graham and Wright (UCL and Birkbeck College 2007)*

Linearised RBC model with idiosyncratic productivity shocks

- ▶ Individuals observe own wage and return on aggregate capital
- ▶ Calibrate relative variance of idiosyncratic and common productivity
  - ▶ Calibration:  $\sigma_a = 0.7\%$  and  $\sigma_a = 4.9\%$
  - ▶ Small relative variance of idiosyncratic productivity shocks imply own wage a precise measure of aggregate
- ▶ Non-zero idiosyncratic productivity shocks invert initial response to aggregate productivity shocks

**Figure 1: Response of consumption to a 1% positive innovation to aggregate productivity<sup>14</sup>**



## *Dynamic Pricing and Imperfect Common Knowledge, Nimark (JME 2008)*

New Keynesian Model with idiosyncratic marginal cost

- ▶ Price setters observe own marginal cost and lagged aggregates
- ▶ Explains inflation inertia and large individual price changes and matches average price durations

**Table 1**

**Simulated data and estimates of the HNK Phillips Curve**

$\sigma_{\varepsilon}^2/\sigma_{mc}^2$	$\mu_b$	$\mu_f$	$\kappa$
1/10	0.19	0.81	0.024
1/2	0.34	0.66	0.013
1	0.41	0.59	0.010

**Table 2**

**Magnitude of individual and aggregate price changes**

$\sigma_\varepsilon^2 / \sigma_{mc}^2$	$E  \Delta p_t(j)  / E  \Delta p_t $	
	$\theta = 0.8$	$\theta = 0.9$
1/10	6.3	13.4
1/2	6.6	17.5
1	7.1	20.4

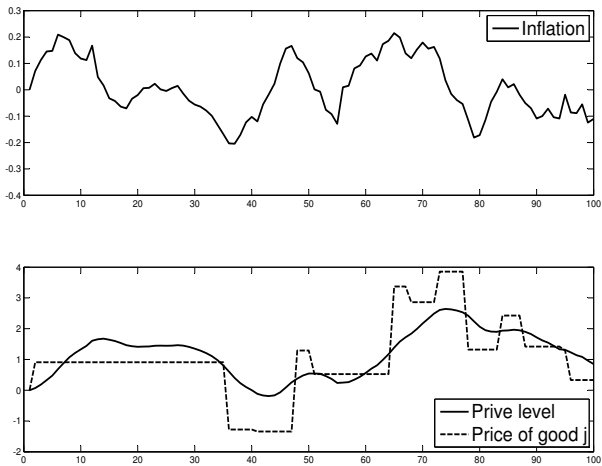


Figure: Inflation, price level and example individual price path

## Summing up:

Hard to make claims about limits of information available to agents

Perhaps we can at least provide bounds on how uninformed agents in our models can be by checking how informative observations from the markets they participate in are