ECONOMETRIC METHODS II: TIME SERIES

MIDTERM JUNE 23 2009

There are 4 questions. Everybody should answer Question 1. Choose two questions from Questions 2 - 4 so that you answer three questions in total. Each answered question is worth a maximum of 10pts. You have 2 hours.

Question 1

Consider the VARMA(2,2) model

$$\begin{aligned} y_t &= \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t + \Psi_1 \varepsilon_{t-1} \\ E\left(\varepsilon_t \varepsilon_t'\right) &= s \cdot I \end{aligned}$$

where y_t is an $(n \times 1)$ vector, s is scalar and I is the identity matrix.

a) Find the state space representation

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t \\ Z_t &= DX_t + \mathbf{v}_t \\ E\left(\mathbf{u}_t \mathbf{u}_t'\right) &= I \\ E\left(\mathbf{v}_t \mathbf{v}_t'\right) &= \Sigma_{vv} \end{aligned}$$

with the vector of observables $Z_t = y_t$.

b) Find an operational expression for the unconditional variance of y_t based on the state space matrices A, C, D and Σ_{vv}.

c) Find the impulse response function

$$\frac{\partial y_{t+s}}{\partial \varepsilon_{1t}}$$

as a function of A, C and D where ε_{1t} is the first element in the vector $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} & \varepsilon_{2t} & \cdots & \varepsilon_{nt} \end{bmatrix}'$
.

Consider the structural VAR

$$\begin{aligned} A_0 y_t &= A_1 y_{t-1} + A_2 y_{t-2} + \mathbf{u}_t \\ E\left(\mathbf{u}_t \mathbf{u}_t'\right) &= I \end{aligned}$$

where y_t is a an $(n \times 1)$ vector.

d) Find the reduced form and show how it can be estimated.

e) If $n = 4$, how many restrictions do you need to identify A_0?

Question 2 Cointegration

a) Define stationarity.
b) For the system

$$\begin{aligned} y_{1t} &= \gamma y_{2t} + u_{1t} \\ y_{2t} &= y_{2t-1} + u_{2t} \end{aligned}$$

show that the y_1 and y_2 are cointegrated (u_{1t} and u_{2t} are white noise processes). Find a cointegrating vector that is not $\left[\begin{array}{cc} 1 & -\gamma \end{array}\right]$.
c) For the system above, show how and why OLS can be used to consistently estimate the parameter γ.
d) The variables $\mathbf{y}_t$ in (1) can be represented as a VAR(3)

$$\mathbf{y}_t = \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi_2} \mathbf{y}_{t-2} + \mathbf{\Phi_3} \mathbf{y}_{t-3} + \varepsilon_t \tag{0.1}$$

and are cointegrated. Find the Vector Error Correction (VECM) representation of $\mathbf{y}_t$.
e) Explain intuitively why a VAR in a finite number of lagged first differences is misspecified if the variables are cointegrated.

Question 3 The Kalman filter

Consider the scalar state space system

$$\begin{aligned} x_t &= \rho x_t + u_t \\ z_t &= d x_t + v_t \\ \left[\begin{array}{c} u_t \\ v_t \end{array}\right] &\sim N\left(0, \left[\begin{array}{cc} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{array}\right]\right) \\ x_{0|0} &= \overline{x}_0 \\ E\left(\overline{x}_0 - x_0\right)^2 &= p_{0|0} \end{aligned}$$

a) Find the Kalman gain k_t such that $x_{t|t}$ given by

$$x_{t|t} = \rho x_{t-1|t-1} + k_t \left[z_t - \rho x_{t-1|t-1}\right]$$

is the linear minimum variance estimate of x_t conditional on $\overline{x}_0$ and the history of z_t up to period t. Denote the prior uncertainty $p_{t|t-1} \equiv E\left(x_{t|t-1} - x_t\right)^2$.
b) What is k_t if $\sigma_v^2 = 0$? Interpret.
c) What is k_t if $\sigma_u^2 = \infty$? Interpret.
d) What is k_t if $d = 0$? Interpret.
e) For arbitrary $p_{0|0}$, d and σ_v^2,what are the upper and lower bounds of $p_{t|t-1}$?Why?

Question 4 Numerical Maximization

(1) Consider the model

$$\begin{aligned} y_t &= \rho y_{t-1} + u_t \\ u_t &\sim N(0, \sigma_u^2) \end{aligned} \tag{0.2}$$

(a) For the AR(1) model (2), what are the parameters that we can estimate using maximum likelihood, i.e. what is the parameter vector that we often denote θ?
(b) Describe the steepest ascent method to find the MLE $\widehat{\theta}$.
(c) Describe the grid search method to find the MLE $\widehat{\theta}$
(d) Describe a simulated annealing algorithm that can be used to find the MLE $\widehat{\theta}$
(e) What are the advantages and disadvantages of simulated annealing, relative to grid search and steepest ascent methods?