IRFs and Variance decompositions of SVARs

June 3, 2009

A simple SVAR(1) example

1. Simulate data
2. Estimate reduced form VAR
3. Impose identifying assumptions
4. Compute impulse responses
5. Compute variance decomposition

Code and slides available at
http://www.kris-nimark.net/TS_UPF_2009.html

Define true model

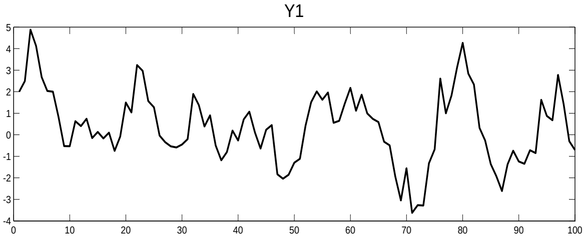
```
A0=[1,0;
  -.2,1];

 A1=[0.9,-.05;
   .2,.75;];

C=inv(A0);
FI=inv(A0)*A1;
OMEGA=inv(A0)*(inv(A0))';
```

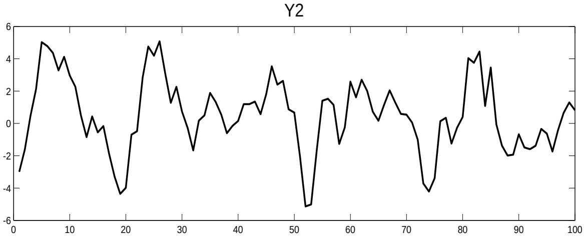
Simulate data

```
T=100;
Y=zeros(2,T);
Y(:,1)=[2;-3;];
U=zeros(2,T);

for tt=2:T;
    U(:,tt)=randn(2,1);
    Y(:,tt)=FI*Y(:,tt-1) + C*U(:,tt);
end
```

Plot time series

```
figure(1)

subplot(2,1,1);
plot(Y(1,:),'k','linewidth',2);
title('Y1','fontsize',18);

subplot(2,1,2);
plot(Y(2,:),'k','linewidth',2);
title('Y2','fontsize',18);
```


Y1
5
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40
50
60
70
80
90
100


Y2
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
60
70
80
90
100

Estimate reduced form

```
y=Y(:,2:end);
y1=Y(:,1:end-1);
FIhat=y*y1'*inv(y1*y1');
OMEGAhat=inv(T-1)*(y-FIhat*y1)*(y-FIhat*y1)';
```

$$\widehat{\Phi} = \begin{bmatrix} 0.996 & -0.22 \\ 0.50 & 0.68 \end{bmatrix} \ldots \left(\Phi = \begin{bmatrix} 0.9 & -0.2 \\ 0.38 & 0.71 \end{bmatrix} \right)$$

$$\widehat{\Omega} = \begin{bmatrix} 0.99 & 0.38 \\ 0.38 & 1.14 \end{bmatrix} \ldots \left(\Omega = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1.04 \end{bmatrix} \right)$$

Find estimated structural form using a12=0

```
Chat=chol(OMEGAhat)';
A0hat=inv(Chat);
A1hat=inv(Chat)*FIhat;
```

$$\widehat{A}_0 = \begin{bmatrix} 1.00 & 0 \\ -0.38 & 1.00 \end{bmatrix} \ldots \left(A_0 = \begin{bmatrix} 1.00 & 0 \\ -0.2 & 1.00 \end{bmatrix} \right)$$

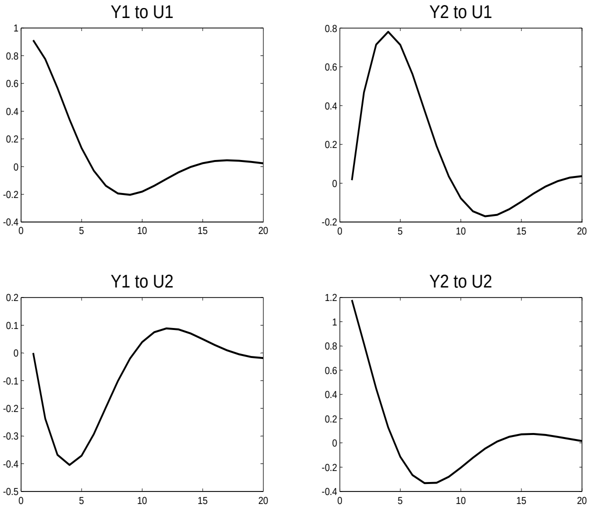
$$\widehat{A}_1 = \begin{bmatrix} 0.99 & -0.21 \\ 0.12 & 0.76 \end{bmatrix} \ldots \left(A_1 = \begin{bmatrix} 0.9 & -0.2 \\ 0.2 & 0.75 \end{bmatrix} \right)$$

Compute impulse response to structural shocks

```
S=20;
IRF1=zeros(2,S);
IRF2=zeros(2,S);

for ss=1:S;
  IRF1(:,ss)=FIhat ^ (ss-1)*Chat(:,1);
  IRF2(:,ss)=FIhat ^ (ss-1)*Chat(:,2);
end
```

Plot impulse response to structural shocks

```
figure(2)
subplot(2,2,1);

plot(IRF1(1,:),'k','linewidth',2);
title('Y1 to U1','fontsize',18);
subplot(2,2,2);

plot(IRF1(2,:),'k','linewidth',2);
title('Y2 to U1','fontsize',18);
subplot(2,2,3);

plot(IRF2(1,:),'k','linewidth',2);
title('Y1 to U2','fontsize',18);
subplot(2,2,4);

plot(IRF2(2,:),'k','linewidth',2);
title('Y2 to U2','fontsize',18);
```

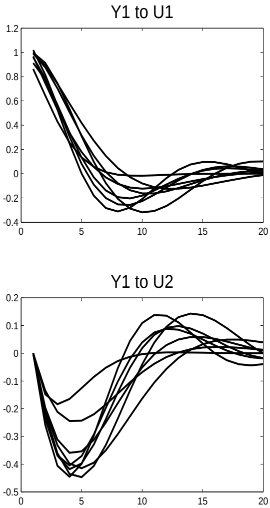


Y1 to U1
Y2 to U1
Y1 to U2
Y2 to U2

A warning:

Sample variance of impulse responses for typical length of sample can be quite large

- $T = 100 \Rightarrow 25$ years of quarterly data
- Re-sampling keeping coefficients fixed can illustrate this


Y1 to U1
Y2 to U1
Y1 to U2
Y2 to U2

Compute unconditional variance

```
diff=1;
tol=1e-6;
SIGYYst=zeros(2,2);

while diff > tol;
  SIGYY=FI*SIGYYst*FI'+C*C';
  diff=max(max(abs(SIGYY-SIGYYst)));
  SIGYYst=SIGYY;
end
```

Compute variance due to individual shocks

```
 diff=1;
SIGYY1st=zeros(2,2);
while diff > tol;
  SIGYY1=FI*SIGYY1st*FI'+C(:,1)*C(:,1)';
  diff=max(max(abs(SIGYY1-SIGYY1st)));
  SIGYY1st=SIGYY1;
end
diff=1;
SIGYY2st=zeros(2,2);
while diff > tol;
  SIGYY2=FI*SIGYY2st*FI'+C(:,2)*C(:,2)';
  diff=max(max(abs(SIGYY2-SIGYY2st)));
  SIGYY2st=SIGYY2;
end
```

Compute fraction of variances variance due to individual shocks

```
frac1=diag(SIGYY1)./diag(SIGYY);
frac2=diag(SIGYY2)./diag(SIGYY);
```

In our example we get

$$\begin{aligned} \texttt{frac1} &= \begin{bmatrix} 0.79 \\ 0.56 \end{bmatrix} \\ \texttt{frac2} &= \begin{bmatrix} 0.21 \\ 0.44 \end{bmatrix} \end{aligned}$$