

# Temes de Macro: Macro Modeling at Central Banks

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# The Plan

Today:

- ▶ The Lucas Critique and its consequences
- ▶ Dynamic Stochastic General Equilibrium (DSGE) Models

Reading package available at <http://www.kris-nimark.net/>

## Last time:

- ▶ Inflation targeting
- ▶ What makes a model useful?
- ▶ Identification
- ▶ Structural VARs and identification of policy shocks

# Structural VARs

Pros:

- ▶ Imposes few assumption
- ▶ Allows for very flexible dynamics
- ▶ Easy to estimate

Cons:

- ▶ Hard to interpret
- ▶ Hard to combine with other information
- ▶ Perhaps uses too little economic theory?

Today we will discuss a class of models that remedies some of the weaknesses of SVARs

## Academic research and macro models

In the 1970s a series of papers by Lucas (1972), (1973) and (1976) and Sargent (1972) and Sargent and Wallace (1975) had a great impact on macro modeling.

- ▶ Introduced micro founded general equilibrium models of an “entire” economy
- ▶ These models had strong normative implications: Government intervention usually counterproductive
- ▶ The normative implications have not survived, but the style of modeling became standard among academics in the 1980s.

# The Lucas Critique

Lucas (1976) critique of statistical models:

- ▶ Statistical models are not robust to many of the policy evaluation exercises that they were used for
- ▶ When policy change, the correlations in the past data may not hold in the future

The remedy:

- ▶ Build models with micro foundations, based on preferences and technology that are presumed to be invariant to changes in policy

## “Modern macro models” (Kocherlakota)

1. Specifies constraints for households, technologies for firms and resource constraints for the whole economy
2. Specifies household preferences and firm objectives
3. Assumes forward looking behavior
4. Includes stochastic processes
5. Are models of the entire economy: No prices are exogenous

# (Bayesian) DSGE models

- ▶ Micro founded
  - ▶ Dynamics of aggregate economy constructed from optimizing behavior of individuals and their budget constraints
- ▶ Rational expectations
  - ▶ “Rational” simply means model consistent
  - ▶ The benchmark equilibrium concept
- ▶ Bayesian estimation
  - ▶ Treat parameters as random variables, just like the shocks
  - ▶ Combine prior and sample information in a transparent manner

## The strategy

- ▶ Specify problems of representative agent and firm and the technologies at their disposal
- ▶ Log-linearize model to find approximate first order conditions and budget constraints
- ▶ Estimate using likelihood based methods
  - ▶ This is where the Bayesian component will come in

# The building blocks of a prototypical DSGE model

- ▶ A representative agent that consumes and supply labor
- ▶ A representative firm that sets prices to maximize profit
- ▶ A central bank that sets nominal interest rates
- ▶ Exogenous shocks (or “wedges”)

## Example: A small open economy model

Model of Australia:

- ▶ Trade is important
- ▶ Australia is small enough to not affect the world economy

Based on Nimark (2009)

## The representative agent

Period  $t$  utility of agent  $i \in (0, 1)$  is given by

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s U(C_{t+s}(i), N_{t+s}(i)) \right\}$$

where

$$U(C_t(i), N_t(i)) = \exp(\varepsilon_t^c) \frac{(C_t(i)H_t^{-\eta})^{1-\gamma}}{(1-\gamma)} - \frac{N_t(i)^{1+\varphi}}{1+\varphi}$$

and  $C_t$  is consumption,  $N_t$  is supplied hours of labor and  $H_t$

$$H_t = \int C_{t-1}(i) di$$

is a habit stock. The budget constraint is given by

$$B_{t+1} + B_{t+1}^* + C_t - \frac{\psi}{2} B_t^{*2} = Y_t + (e^{v_t^{px}} - 1)X_t + R_t \frac{P_{t-1}}{P_t} B_t + \frac{S_t P_{t-1} R_t^* B_t^*}{S_{t-1} P_t}$$

## The decisions made by the representative agent

- ▶ How much to consume and how much to save (Euler-equation)

$$U_c(C_t) = \beta E_t R_t \frac{P_t U_c(C_{t+1})}{P_{t+1}}$$

- ▶ What to consume: Imported or domestically produced goods?

$$C_t^m = C_t \exp(v_t^m) \left( \frac{P_t^m}{P_t} \right)^{-\delta}$$

- ▶ How to save: Domestic or foreign currency denominated bonds (UIP condition)

$$R_t = (\exp v_t^s) E_t \frac{R_t^*}{\psi B_t^*} \frac{S_t}{S_{t+1}}$$

- ▶ How much to work:

$$\frac{W_t}{P_t} U_c(C_t) = N_t^\varphi$$

## Production of goods

Output in firm  $j \in (0, 1)$  is produced using labor as the sole input

$$Y_t(j) = \exp(a_t)N_t(j)$$

where  $a_t$  is an exogenous labor augmenting technology.

## Price setting

Prices are reset only infrequently (with  $prob = 1 - \theta$ ) and a fraction of firms use a rule of thumb that sets current price equal to last periods optimal price. This results in a (linearized) Phillips curve

$$\pi_t^d = \mu_f^d E_t \pi_{t+1}^d + \mu_b^d \pi_{t-1}^d + \lambda mc_t^d + \varepsilon_t^\pi$$

Monopolistic competition: Each firm produces a unique good and has some market power.

## The Rest of the World

Australia is small: We can treat rest of world as exogenous

$$\begin{bmatrix} y_t^* \\ \pi_t^* \\ i_t^* \end{bmatrix} = M \begin{bmatrix} y_{t-1}^* \\ \pi_{t-1}^* \\ i_{t-1}^* \end{bmatrix} + \varepsilon_t^*$$

Still, demand from the rest of the world for Australian exports depends on relative prices

$$X_t = (\exp v_t^x) \left( \frac{P_t^d}{P_t^*} \right)^{-\delta^x} Y_t^*$$

## Monetary policy

Monetary policy is represented by a simple Taylor-type rule

$$i_t = \phi_y y_{t-1} + \phi_\pi \pi_{t-1} + \phi_i i_{t-1} + \varepsilon_t^i$$

Can be viewed as a reduced form representation of an optimizing central bank and fits the data quite well.

## Log-linearizing

First order conditions and budget constraints etc can be put in the form

$$A_0 \begin{bmatrix} X_{1t} \\ E_t X_{2,t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{1t-1} \\ X_{2t} \end{bmatrix} + C_1 \mathbf{u}_t$$

where  $X_{1t}$  is vector containing the state variables and  $X_{2t}$  is a vector containing the forward looking endogenous variables.

- ▶ State variables: Exogenous shocks and predetermined endogenous variables

By imposing rational expectations, we can solve model so that to get it in the form

$$\begin{aligned} X_{1t} &= AX_{1t-1} + C\mathbf{u}_t \\ X_{2t} &= GX_{1t} \end{aligned}$$

where  $A$ ,  $C$  and  $G$  will be (usually non-linear) functions of the "deep" parameters  $\Theta = \{\gamma, \eta, \varphi, \dots\}$  describing preferences, technology and exogenous processes.

## Estimating the model

The model in state space form

$$\begin{aligned}X_{1t} &= AX_{1t-1} + C\varepsilon_t \\Z_t &= DX_{1t} + \mathbf{e}_t\end{aligned}$$

$Z_t$  is a vector of observable time series and  $D$  is a mapping between the state and the observable variables.

This system can be estimated using likelihood based methods that combine prior and sample information.

## The data

$Z_t =$

trade weighted G7 output

trade weighted G7inflation

average of US, Japanese and German/euro interest rates

the cash rate (RBA policy instrument)

the nominal exchange rate

trimmed mean quarterly CPI inflation

GDP, non-farm GDP

market sector GDP

exports as share of GDP

the terms of trade (defined as the price of exports over the price of imports)

labor productivity

## Bayesian estimation

Combining prior and sample information

Prior information:

- ▶ We may have independent evidence on risk aversion, price stickiness, average real interest rates etc

Sample information

- ▶ The log likelihood function

$$\ln \mathcal{L}(Z | \Theta) = -.5 \sum_{t=0}^T [p \ln(2\pi) + \ln |\Omega| + u_t' \Omega^{-1} u_t]$$

tells us how likely the data  $Z$  is to have been generated by the model parameterized with  $\Theta$ .

Bayesian methods combine prior and sample information into a posterior estimate of the parameter vector  $\Theta$ .

$$\arg \max_{\theta} p(\theta | Z) = \arg \max_{\theta} \mathcal{L}(Z | \Theta) p(\theta | Z)$$

**Table 1: Prior and posterior distributions of parameters**

Parameter	Distribution	Prior		Posterior	
		Mode	S.d.	Mode	S.d.
Households and firms					
$\gamma$	normal	3	0.44	2.97	0.30
$\eta$	normal	2	0.66	1.48	0.13
$\varphi$	normal	2	0.44	1.35	0.31
$\omega$	beta	0.3	0.10	0.24	0.08
$\delta$	normal	1	0.10	0.86	0.09
$\delta^x$	normal	1	0.10	0.15	0.06
$\theta$	beta	0.75	0.04	0.89	0.01
$\theta^m$	beta	0.75	0.04	0.90	0.01
$\psi$	normal	0.01	0.02	0.07	0.02

**Table 2: Prior and posterior distributions of parameters**

Parameter	Distribution	Prior		Posterior	
		Mode	S.d.	Mode	S.d.
		Taylor rule			
$\phi_y$	normal	0.5	0.25	0.02	0.01
$\phi_\pi$	normal	1.5	0.29	0.41	0.04
$\phi_i$	beta	0.5	0.25	0.87	0.03
$\sigma_i^2$	uniform	$[0, \infty)$		$7.58 \times 10^{-7}$	$1.86 \times 10^{-7}$

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**Table 3: Prior and posterior distributions of parameters**

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Parameter	Distribution	Prior		Posterior	
		Mode	S.d.	Mode	S.d.
Exogenous persistence					
$\rho_a$	beta	0.5	0.28	0.71	0.05
$\rho_s$	beta	0.5	0.28	0.81	0.08
$\rho_{px}$	beta	0.5	0.28	0.81	0.05
$\rho_x$	beta	0.5	0.28	0.90	0.07
$\rho_m$	beta	0.5	0.28	0.80	0.05

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**Table 4: Prior and posterior distributions of parameters**

Parameter	Distribution	Prior		Posterior	
		Mode	S.d.	Mode	S.d.
Exogenous shock variances					
$\sigma_a^2$	uniform	$[0, \infty)$		$9.13 \times 10^{-5}$	$2.30 \times 10^{-5}$
$\sigma_v^2$	uniform	$[0, \infty)$		$2.02 \times 10^{-3}$	$2.59 \times 10^{-3}$
$\sigma_c^2$	uniform	$[0, \infty)$		$1.79 \times 10^{-5}$	$5.81 \times 10^{-6}$
$\sigma_\pi^2$	uniform	$[0, \infty)$		$2.70 \times 10^{-5}$	$6.92 \times 10^{-5}$
$\sigma_{p_x}^2$	uniform	$[0, \infty)$		$6.82 \times 10^{-5}$	$3.39 \times 10^{-5}$
$\sigma_x^2$	uniform	$[0, \infty)$		$4.89 \times 10^{-5}$	$8.84 \times 10^{-5}$
$\sigma_m^2$	uniform	$[0, \infty)$		$2.12 \times 10^{-5}$	$5.51 \times 10^{-6}$
$\sigma_i^2$	uniform	$[0, \infty)$		$7.58 \times 10^{-7}$	$1.86 \times 10^{-7}$

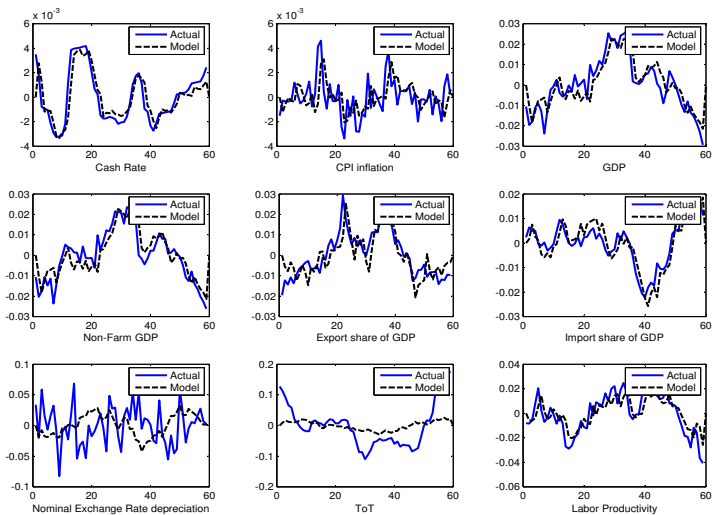


Figure: Actual and model's one period ahead forecasts

## What we can do with the estimated model

- ▶ Impulse response functions
- ▶ Density forecasts
- ▶ Historical decompositions

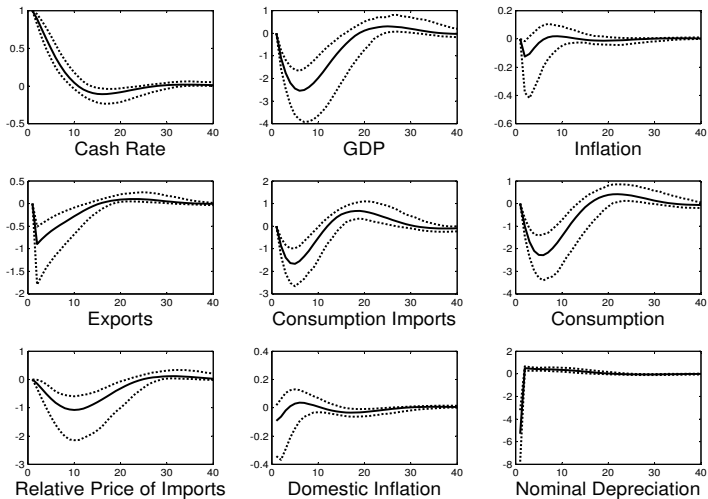


Figure: Monetary Policy Shock

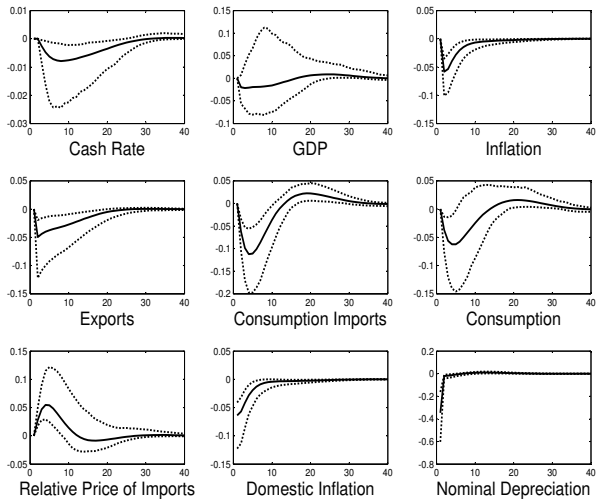
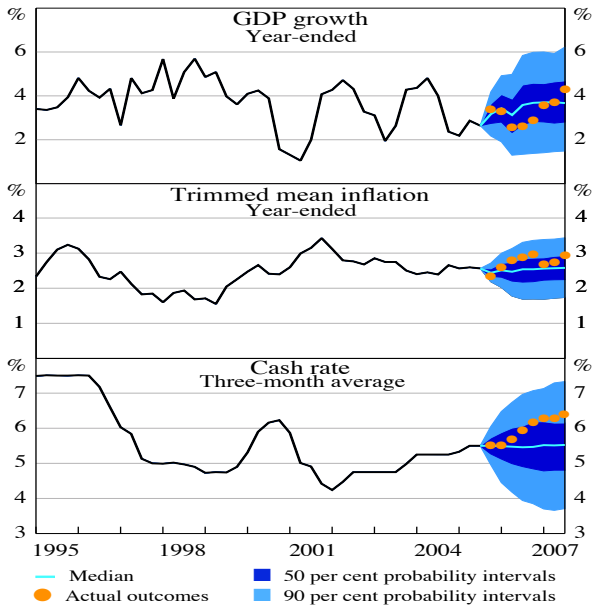


Figure: Productivity Shock



## More things one can do with DSGE models

### Utility based welfare measures

- ▶ Use utility functions to derive a n optimal policy from the perspective of the representative agent

### Counterfactual scenarios

- ▶ If we have estimated truly structural parameters (taste, technology etc) we can do counterfactual scenarios that are robust to the Lucas critique

## DSGE models in the policy process

- ▶ Corresponds more closely to how economists think about the economy which makes it easier to incorporate into decision process
- ▶ Information from sector specialists can more easily be incorporated into forecasts

## DSGE models: gaps to plug

- ▶ Unemployment
- ▶ Heterogeneity
- ▶ Financial sector
- ▶ Rational expectations
- ▶ And by the way, “shocks” what are they really?

## Summing up:

### SVARs

- ▶ Uses only weak restrictions
- ▶ Flexible dynamics
- ▶ Not always easy to interpret

### DSGE models

- ▶ Impose a lot of restrictions on the data
  - ▶ Good, since it allows us to interpret model using economic reasoning
  - ▶ Bad, if it means water boarding the data so that it tells us what we want to hear
- ▶ Can be used for counterfactual scenarios

Both approaches has pros and cons and what one prefer is often a matter of personal taste or the question at hand.

## Readings:

Nimark (2009) : Presents and estimates a simple DSGE model

Kocherlakota (2010): Discusses the state of macro modeling for policy purposes