TOPICS IN MACROECONOMICS: MODELLING INFORMATION, LEARNING AND EXPECTATIONS

HOMEWORK 1
FALL 2009

Answer all questions using MatLab. Write up your answers and how they were derived and submit in pdf form together with supporting m-files by Tuesday November 24 to knimark@crei.cat. Where possible, please use notation established in lecture notes.

Question 1

Consider the stochastic difference equation

$$X_t = AX_{t-1} + C\mathbf{u}_t$$

where X_t is an 2×1 vector of random variables, $\mathbf{u}_t$ is an 2×1 vector of i.i.d. shocks with unit variance, i.e. $E\left[\mathbf{u}_t\mathbf{u}_{t+s}'\right] = I$ if $s = 0$ and $\mathbf{0}$ otherwise. A and C are (2×2 and 2×2 respectively) coefficient matrices given by

$$A = \begin{bmatrix} .8 & .1 \\ 0 & .3 \end{bmatrix}, \quad C = I$$

(a) Compute the unconditional variance $\Sigma_{xx} \equiv E\left[X_t X_t'\right].$

(b) Compute the covariance $\Sigma_{XY} \equiv E\left[X_t Y_t'\right]$ where $Y_t = BX_t$ and $B = \begin{bmatrix} 1 & 1 \end{bmatrix}.$

(c) Find a K such that $KY_t = E\left(X_t \mid Y_t\right).$ What is the error variance $E\left[\left(X_t - KY_t\right)\left(X_t - KY_t\right)'\right]$?

(d) The rank of the matrix G is 2. What is $E\left(X_t \mid Q_t\right)$ if $Q_t = GX_t$? What is the rank of B?

Question 2

Consider the state space system

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t \\ Z_t &= DX_t + \mathbf{v}_t \end{aligned}$$

where $X_{t,}\mathbf{u}_t, A$ and C are the same as in Question 1 and $D = B$ and $\mathbf{v}_t$ is a measurement error with variance $\sigma_v^2 = 0$.

(a) Compute the steady state Kalman gain and the steady state prior and posterior error covariances $K_t, P_{t|t-1}$ and $P_{t|t}$ (where steady state refers to $t \to \infty$).

(b) Compare $P_{t|t}$ from 2 (a) with your answer to question 1 (c). Interpret.

(c) Redo 2 a) but $\sigma_v^2 = 1000$. Compare with your answers to questions 1 (a) and 1 (b).

(d) Compute the steady state Kalman gain with

$$C = \begin{bmatrix} 5 & 0 \\ 0 & 0.1 \end{bmatrix}$$

and discuss how and why K_t changes.