TOPICS IN MACROECONOMICS: MODELLING INFORMATION, LEARNING AND EXPECTATIONS

HOMEWORK 2 FALL 2010

Answer all questions using MatLab. Write up your answers and how they were derived and submit in pdf form together with supporting m-files by Friday December 10 to knimark@crei.cat. Where possible, please use notation established in lecture notes.

Question 1 Higher order expectations

Agents are indexed by $j \in (0,1)$ and estimate the unobservable persistent process

$$\begin{aligned} \theta_t &= 0.95\theta_{t-1} + v_t \\ v_t &\sim N(0,1) \end{aligned}$$

In period t agent j observes the unbiased but noisy signal $s_t(j)$ of the true value of θ_t

$$\begin{aligned} s_t(j) &= \theta_t + \eta_t(j) \\ \eta_t(j) &\sim N(0,1) \ \forall \ j \end{aligned}$$

Notation:

$$\begin{aligned} \theta_t^{(0)} &\equiv \theta_t \\ \theta_t^{(k)} &\equiv \int E\left[\theta_t^{(k-1)} \mid I_t(j)\right] \ dj \\ \theta_t^{(0:k)} &= \left[\begin{array}{cccc} \theta_t^{(0)} & \theta_t^{(1)} & ... & \theta_t^{(k)} \end{array} \right]' \end{aligned}$$

a) Use common knowledge of rationality to find a law of motion for $\theta_t^{(0:\overline{k})}$ for $\overline{k} = 3$

b) Compute $E\left[\theta_t^{(0:\overline{k})}\theta_t'^{(0:\overline{k})}\right]$. Plot the diagonal elements and discuss.

c) The endogenous variable y_t is given by

$$y_t = \mathbf{a}_{\overline{k}}\theta_t^{(0:\overline{k})}$$

but y_t is not observed by agents. The elements of $\mathbf{a}_k$ are given by

$$\begin{aligned} \mathbf{a}_{\overline{k}} &\equiv \left[\begin{array}{cccc} a_0 & a_1 & \cdots & a_{\overline{k}} \end{array} \right] \\ a_k &= (0.5)^k \end{aligned}$$

Compute an impulse response for y_t to a shock to θ_t and $\overline{k} = 3$.

d) Redo c) with $\overline{k} = 4$, $\overline{k} = 5$, $\overline{k} = 6$. Discuss.

e) Redo d) with $a_k = (0.98)^k$ Discuss.

f) Redo e) with $\eta_t(j) \sim N(0,10) \ \forall \ j$. Discuss.

Question 2 Learning and Bounded Rationality

Consider the Cob-Web model

$$\begin{aligned} p_t &= \mu + \alpha E_{t-1} p_t + \delta w_{t-1} + \eta_t \\ w_t &= \rho w_{t-1} + u_t \end{aligned}$$

The REE of the Cob-Web model is given by

$$p_t = \frac{\mu}{1-\alpha} + \frac{\delta}{1-\alpha} w_{t-1} + \eta_{t-1} \quad (REE)$$

a) Simulate the cob web model for 100 periods under recursive least squares/decreasing gain learning when agents fit a perceived law of motion that nests the REE. Use the parameter values $\mu = 1, \alpha = 0.9, \delta = .2,\ \rho = 0.8$. Report the evolution of the price level p_t and the evolution of the parameters (including the chosen starting values) in the perceived law of motion of agents.

b) Simulate the cob web model for 100 periods under constant gain learning when agents fit a perceived law of motion that nests the REE. Discuss differences between decreasing and constant gain learning.

c) Simulate the cob web model for 100 periods under recursive least squares/decreasing gain learning when agents fit the perceived law of motion

$$p_t = a_{t-1} + e_t \quad (PLM)$$

d) Redo 2 a) with $\alpha = 1.5$. Discuss.