MODELLING INFORMATION, LEARNING AND EXPECTATIONS IN MACROECONOMICS

HOMEWORK 1

Answer all questions using MatLab. Write up your answers and submit in pdf form together with supporting m-files by Tuesday February 26. Please use notation established in lecture notes. Extra points can be gained from pointing out instances where that notation is inconsistent.

Question 1

a) Solve the model

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - \overline{y}_t) + \varepsilon_t \tag{0.1}$$

$$y_t = E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) \tag{0.2}$$

$$i_t = \phi \pi_t \tag{0.3}$$

$$\overline{y}_t = \rho \overline{y}_{t-1} + u_t \tag{0.4}$$

by replacing expectations terms with linear projections on π_t and y_t. Use the following parameter values $\beta = .99, \kappa = .3, \sigma = 2, \phi = 1.5$ and $\rho = .8$ Set all variances to unity.

b) Are the expectations errors $E_t\pi_{t+1} - \pi_{t+1}$ and $E_t y_{t+1} - y_{t+1}$ orthogonal to π_t and y_t?

c) Add a demand shock to the model

$$y_t = E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + v_t \tag{0.5}$$

and resolve it, again replacing expectations with projections on π_t and y_t.

d) Are the expectations errors $E_t\pi_{t+1} - \pi_{t+1}$ and $E_t y_{t+1} - y_{t+1}$ orthogonal to π_t and y_t?

e) Are the expectations errors $E_t\pi_{t+1} - \pi_{t+1}$ and $E_t y_{t+1} - y_{t+1}$ orthogonal to π_{t-1} and y_{t-1}? Why or why not? In what sense are, or are not, expectations based on projections on π_t and y_t optimal?

Question 2

Consider the system

$$X_t = A_t X_{t-1} + C_t \mathbf{u}_t \tag{0.6}$$

$$Z_t = D_t X_t + \mathbf{v}_t \tag{0.7}$$

where X_t is an $n \times 1$ vector of random variables, $\mathbf{u}_t$ is an $m \times 1$ vector of i.i.d. shocks with unit variance, i.e. $E\left[\mathbf{u}_t \mathbf{u}'_{t+s}\right] = I$ if $s = 0$ and $\mathbf{0}$ otherwise. A and C are ($n \times n$ and $n \times m$ respectively) coefficient matrices. Z_t is an $(l \times 1)$ vector of observables and D_t is an $(l \times n)$ selector matrix that combines elements of the state X_t into observable variables and $\mathbf{v}_t$ is an $(l \times 1)$ vector of measurement errors with covariance Σ_{vv}.

a) Set $A_t = .5, C_t = 1, D_t = 1, \Sigma_{vv} = 1$ for all t. Use the initial condition $X_{0|0} = 0$ and $E\left(X_0 - X_{0|0}\right)\left(X_0 - X_{0|0}\right)' = 1$. Assume that $\mathbf{u}_t$ and $\mathbf{v}_t$ are Gaussian and plot the sequence of probability density functions that describe the random variable $\left(X_t - X_{t|t-1}\right)$ for $t = 1, 2, 3, 4, 5$. Discuss.

b) Using the same coefficient matrices as in a), compute the innovations sequence $\left\{\widetilde{Z}_t\right\}_{t=1}^{5}$ where $\widetilde{Z}_t = Z_t - E\left[Z_t \mid Z_{t-1}, Z_{t-2}, ..., X_{0|0}\right]$ and $\{Z_t\}_{t=1}^{5} = \{1, 0, -2, -1, 2\}$. Find the covariance matrix $E\left[\begin{array}{ccccc} \widetilde{Z}_5 & \widetilde{Z}_4 & \widetilde{Z}_3 & \widetilde{Z}_2 & \widetilde{Z}_1 \end{array}\right]\left[\begin{array}{ccccc} \widetilde{Z}_5 & \widetilde{Z}_4 & \widetilde{Z}_3 & \widetilde{Z}_2 & \widetilde{Z}_1 \end{array}\right]'$. Discuss.

c) Redo a) but with $D_t = \left[\begin{array}{cc} 1 & 1 \end{array}\right]'$ and $\Sigma_{vv} = I_2$ for t even and $D_t = 1$ and $\Sigma_{vv} = 1$ for t odd. Discuss.

d) Set

$$A_t = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix} \tag{0.8}$$

$$C_t = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \tag{0.9}$$

$$D_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{0.10}$$

$$\Sigma_{vv} = 1 \tag{0.11}$$

for all t. Compute the time invariant prior error covariance $P_{t|t-1} \equiv E\left(X_t - X_{t|t-1}\right)\left(X_t - X_{t|t-1}\right)'$ as $t \to \infty$. Why are the diagonal entries different? Compute the associated time invariant Kalman gain K in

$$X_{t|t} = AX_{t-1|t-1} + K\left(Z_t - DAX_{t-1|t-1}\right) \tag{0.12}$$

e) K in d) is a 2×1 matrix. Why are the elements of K not the same?