TOPICS IN MACROECONOMICS: MODELLING INFORMATION, LEARNING AND EXPECTATIONS

HOMEWORK 1

Answer all questions using MatLab. Write up your answers and how they were derived and submit in pdf form together with supporting m-files by Friday October 10 to knimark@crei.cat. Where possible, please use notation established in lecture notes.

Question 1

Consider the stochastic difference equation

$$X_t = AX_{t-1} + C\mathbf{u}_t$$

where X_t is an 2×1 vector of random variables, $\mathbf{u}_t$ is an 2×1 vector of i.i.d. shocks with unit variance, i.e. $E\left[\mathbf{u}_t \mathbf{u}'_{t+s}\right] = I$ if $s = 0$ and $\mathbf{0}$ otherwise. A and C are (2×2 and 2×2 respectively) coefficient matrices given by

$$A = \begin{bmatrix} .5 & .1 \\ .1 & .5 \end{bmatrix}, \quad C = I$$

Compute:

a) The expected discounted sum $\sum_{s=0}^{\infty} \beta^s E\left[X_{t+s} \mid X_t\right]$ for $X_t = \begin{bmatrix} 1 & 1 \end{bmatrix}'$ and $X_t = \begin{bmatrix} 1 & -1 \end{bmatrix}'$

b) The unconditional variance $\Sigma_{xx} \equiv E\left[X_t X'_t\right]$

c) The unconditional variance $\Sigma_{YY} \equiv E\left[Y_t Y'_t\right]$ where $Y_t = BX_t$ and $B = \begin{bmatrix} 1 & 1 \end{bmatrix}$

Question 2

Consider the state space system

$$X_t = AX_{t-1} + C\mathbf{u}_t$$

$$Z_t = DX_t + \mathbf{v}_t$$

where $X_{t,}\mathbf{u}_t, A$ and C are the same as in Question 1. Z_t is an $(l \times 1)$ vector of observables and D_t is an $(l \times n)$ selector matrix that combines elements of the state X_t into observable variables and $\mathbf{v}_t$ is an $(l \times 1)$ vector of measurement errors with covariance Σ_{vv}.We have the following history of observables $\{Z_t\}_{t=1}^{5} = \{1, 0, -2, -1, 2\}$.

Compute:

a) $X_{t|t} = E\left[X_t \mid Z^t, X_{0|0}\right] : t = 1, 2, ..., 5$ and $D = \begin{bmatrix} 1 & 1 \end{bmatrix}, \Sigma_{vv} = 1, X_{0|0} = \mathbf{0}, P_{0|0} = \mathbf{0}_{2\times2}$. Report by plotting the sequence of the elements of X_t.

b) $P_{t|t-1} = E\left[X_{t|t-1} - X_t\right]\left[X_{t|t-1} - X_t\right]' : t = 1, 2, ..., 5$ and $D, \Sigma_{vv}, X_{0|0}$ and $P_{0|0}$ as in a). Report results by plotting the sequence of diagonal elements (i.e. the variances).

c) Redo a) and b) but with $P_{0|0} = \mathbf{\Sigma}_{xx}$ from your answer to Question 1.

d) Compare the diagonal elements of your sequence $P_{t|t-1}$ from b) and c) with each other and with the diagonal elements of Σ_{xx} and CC'. Discuss.

e) Redo a) and b) with

$$C = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

and discuss how and why the sequences of $X_{t|t}$ and $P_{t|t-1}$ are now different.