Bayesian Robustness and Learning

Topics in Macro   Lecture 9

December 1, 2009

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E-mail list

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Donald Rumsfeld, Feb. 12, 2002, Department of Defense news briefing

Admin stuff

Time and place of midterm: Thursday December 3, Room 20.017 Jaume I

Re-scaling of Homework 1 scores: $X_{new} = 5 + \frac{1}{2} X_{old}$

Bayesian Robustness, Learning and Monetary Policy

Multiple models with different probabilities

- Optimal policy depends on
  - the relative probability of each model
  - the expected loss implied by the different model

We are going to talk mostly about a paper by Cogley and Sargent (RED 2005): *The Conquest of US Inflation: Learning and Robustness to Model Uncertainty*

The Conquest of US Inflation: Learning and Robustness to Model Uncertainty

Tackles an important question: What caused the rise and fall of US inflation over the 1960- 1990 period?
There are many stories:

1. Large shocks in the 70's, nothing policy makers could do
2. Monetary policy did not respect the Taylor principle so inflation expectations could be self fulfilling.
3. Policy makers could not commit to low inflation and the economy suffered from a discretionary policy inflation bias which was later overcome.
4. Policy makers thought there was an exploitable non-vertical Phillips curve but learned over time that this was not the case.

Cogley and Sargent present a version of the 4th hypothesis

Bayesian Robustness, Learning and Monetary Policy

Optimal policy maximizes expected utility (or minimizes expected loss)

- Several (well formulated) models
- Models used to predict consequences of actions
- Probability of each model will (partly) determine weight of model in policy design

Interesting things happen if the policy recommended by the most likely model has catastrophic consequences according to other plausible models

Positive probability of very large losses: An example

Two models:

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = A \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + Br_t + Cu_t$$

$$L_t = \pi_t^2 + y_t^2$$

- Model 1: There is no real effect of predictable policy

$$B_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

- Model 2: Predictable r_t negatively affects next period output.

$$B_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Positive probability of very large losses: An example

Minimize expected loss

$$\min_F E_t \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + y_t^2\right] \quad \text{s.t.}$$

$$\begin{aligned} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} &= A \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + Br_t + Cu_t, \qquad r_t = F \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} \\ p\left(B = B_1\right) &= p_1, \quad p\left(B = B_2\right) = \left(1 - p_1\right) \end{aligned}$$

given p_1

Optimal Policy in Model 1

$$\begin{aligned} X_t &= AX_{t-1} + Br_t + Cu_t \\ r_t &= FX_t \end{aligned}$$

Substitute in policy in state equation

$$X_t = (A + BF)\, X_{t-1} + Cu_t$$

Model 1

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_{11} - f_1 & a_{12} - f_2 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + Cu_t$$

implies that optimal policy is to have

$$F_1^* = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix}$$

Expected inflation is always zero.

Optimal Policy in Model 2

Policy and stability $B_2 = \left[\begin{array}{cc} -1 & -1 \end{array} \right]' \implies$

$$\begin{aligned} X_t &= (A + BF)\, X_{t-1} + Cu_t \\ \left[\begin{array}{c} \pi_t \\ y_t \end{array} \right] &= \left[\begin{array}{cc} 0 & 0 \\ -f_1 & a_{22} - f_2 \end{array} \right] \left[\begin{array}{c} \pi_{t-1} \\ y_{t-1} \end{array} \right] + Cu_t \end{aligned}$$

$eig\,(A + BF) = 0, (a_{22} - f_2)$
Complete stabilization of inflation will therefore only be chosen if $a_{22} - a_{12} < \beta^{-1}$ or $p_1 = 1$, otherwise losses will be infinite.

Cogley and Sargent's story can be understood as a story about the evolution of the policy makers estimates of a_{12}, a_{22} and p_1.

Cogley and Sargent RED (2005)

An account of the evolution of inflation and policy maker's beliefs

- A nice combination of history of thought and empirical economics

Policy makers have three models representing three generations of macro models:

1. Solow-Samuelson: Long run exploitable trade off between inflation and unemployment
2. Solow-Tobin: Short run exploitable trade off, but vertical long run Phillips Curve.
3. Lucas-Sargent: Natural rate, no short run trade off.

Policy makers conduct Bayesian robust optimal policy.

Two aspects of learning

- Evolution of parameter estimates for each model
- Evolution of relative probabilities

The Puzzle

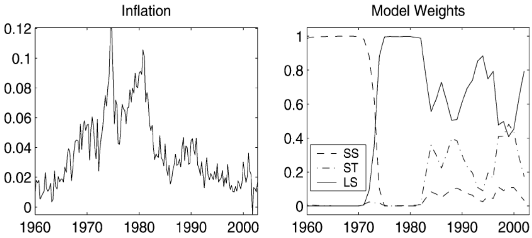

Fig. 1. Inflation and posterior model probabilities.

Expected Losses under the three models

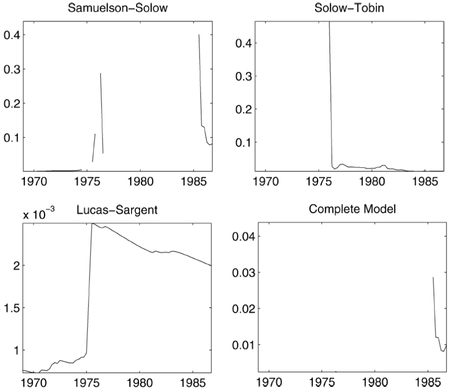

Fig. 3. Expected loss of a zero inflation policy.

The evolution of eigenvalues

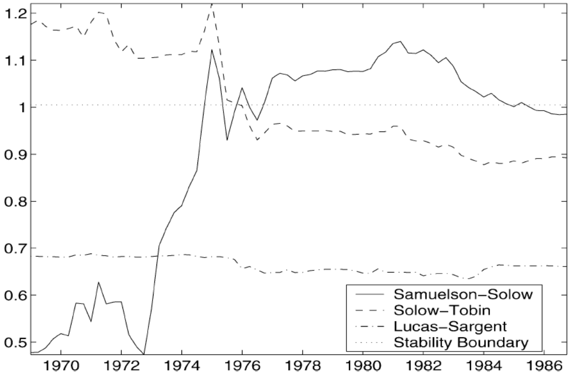

Fig. 4. Dominant eigenvalue under zero inflation.

Summing up

- Probabilities and losses of sub models jointly determines optimal policy
- Delay in reducing inflation was not because of low prob on Lucas-Sargent model but because losses implied by Solow-Samuelson model were large for much longer

Can we understand fiscal policy in the current crisis as Bayesian robust policy?

What follows is a non-exhaustive guide to what you need study before the exam

Lecture 1: Basic methods

- Basic definitions and manipulations of difference equations
- How to find variance-covariance etc of (linear functions of) random variables
- How to use the projection theorem to compute conditional expectations
- how to find conditional expectation errors

Lecture 2: Solving full information rational expectations models

- How the methods discussed relates to methods used to solve imperfect information models
- How to use the method of undetermined coefficients
- Know when and why the three methods give the same answer

Lecture 3: The Kalman filter

- How to derive the scalar Kalman filter
- The properties of the Kalman filter and in what setting it can be applied

Lecture 4: Island models

- The punch-lines of Lucas and Lorenzoni
- How to manipulate the Lucas island model
- A little bit about the modeling strategies of both Lucas and Lorenzoni.

Lecture 5: Private and public information

- Understand the interaction between strategic complementarity and public and private information
- Higher order expectations: What they are and how to compute them
- How to solve Morris and Shin's model
- The punch-line of Morris and Shin + the "Svensson critique"

Lecture 6: Information revealed by markets

- Manipulate the Grossman Stiglitz model
- Understand the intuition behind the "impossibility" result
- Mechanics behind relationship between shocks and information revelation

Lecture 7: Endogenous information choice/rational inattention

- Definition of entropy and mutual information
- How to solve simple optimizing problems with processing contraints
- The main results of Mackowiack and Wiederholt (2009)

Lecture 8: Bounded rationality and learning

- The motivation behind recursive least squares learning
- Constant vs decreasing gain learning
- How to check for convergence of beliefs

Lecture 9: Learning and Bayesian robustness

- The punch-line of Cogley and Sargent (2005)
- The relationship between model probabilities, expected losses and optimal policy

Condensed advice

Basically, read everything linked on course page

Derivations in exam will be similar to those in class